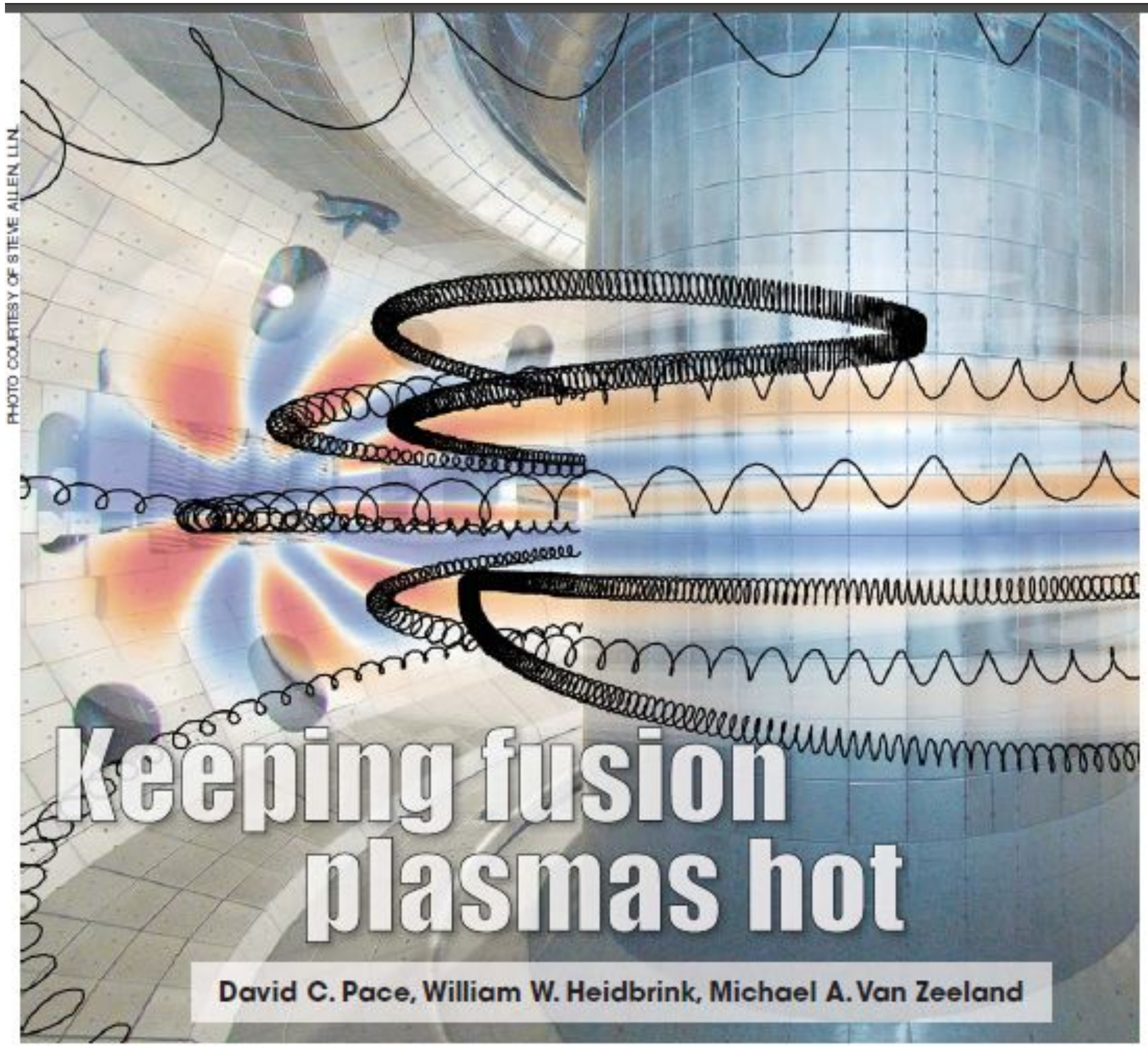
Introduction to Energetic Particle Physics

W.W. (Bill) Heidbrink
UC Irvine



Physics Today, October 2015

Interactions between electromagnetic waves and the most energetic ions in a plasma can perturb the orbits of those ions enough to expel them from the confining magnetic field.

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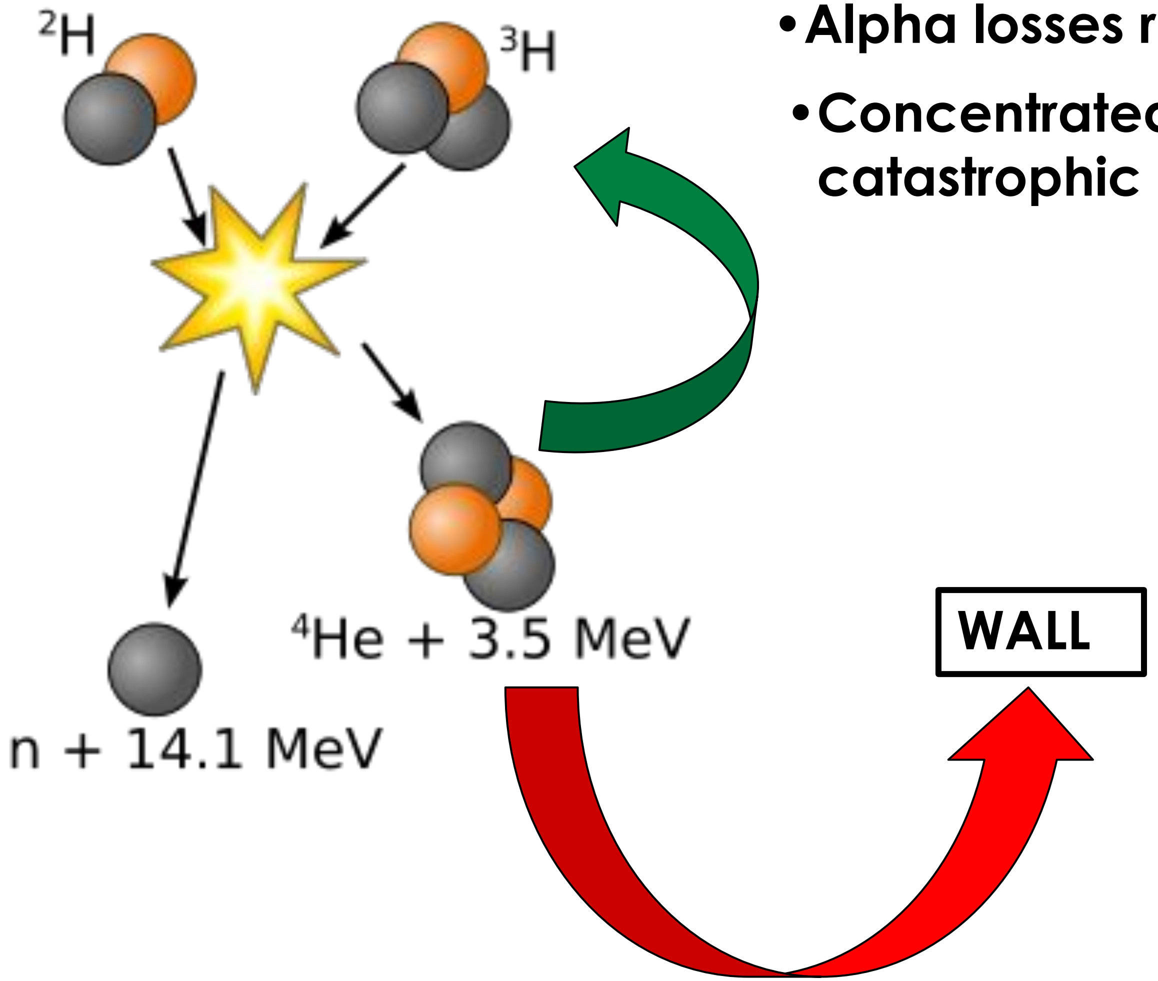
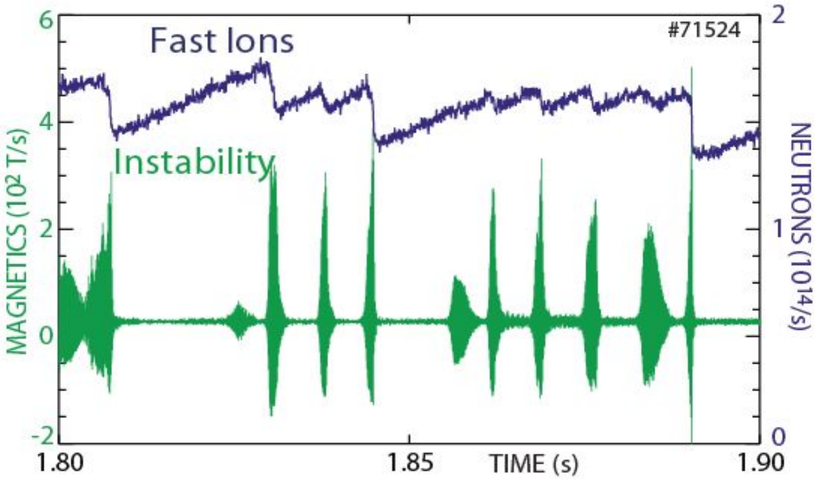
An ignited fusion reactor must confine charged fusion products

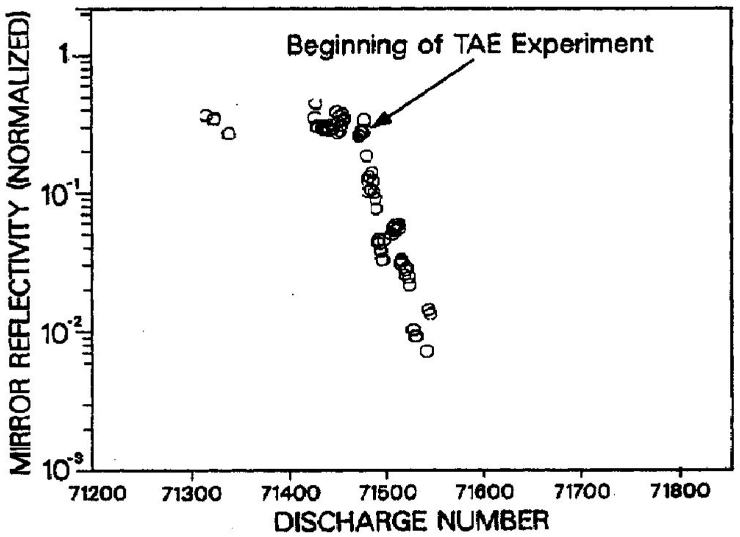


- Alpha losses reduce the heating
- Concentrated losses can be catastrophic

Losses of energetic particles must be controlled in a reactor (and ITER)

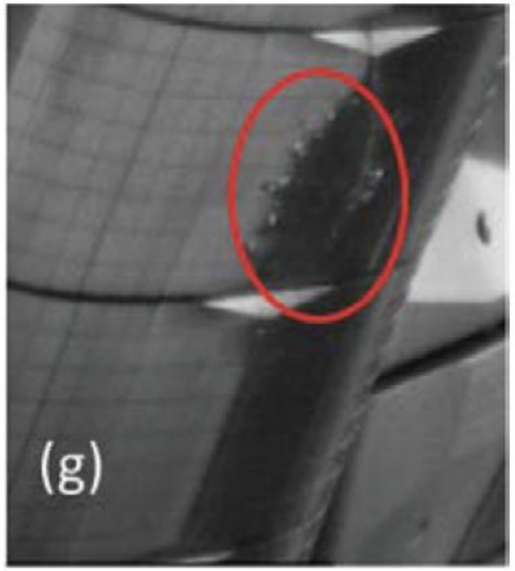
Beam injection into the DIII-D tokamak





Damage

- Carbon coats DIII-D mirrors when escaping fast ions ablate the graphite wall[1]
- Transport of fast ions by Alfvén waves onto unconfined orbits cause a vacuum leak in TFTR[2]
- Runaway electrons damage tiles in JET[3]



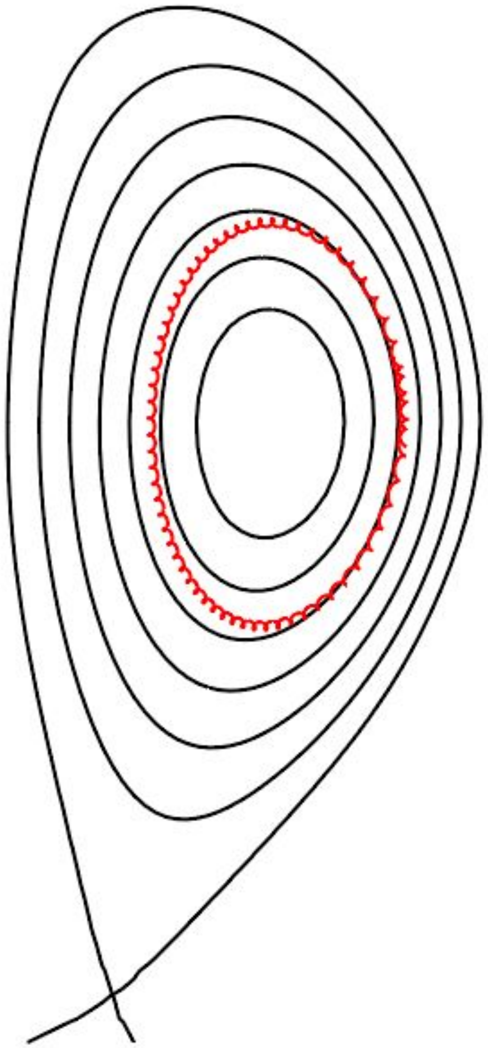
[1]Duong, Nucl. Fusion 33 (1993) 749.

[2]White, Phys. Pl. 2 (1995) 2871.

[3]Reux, Nucl. Fusion 55 (2015) 093013

An Energetic Particle (EP) is a collisionless particle that orbits far from field lines

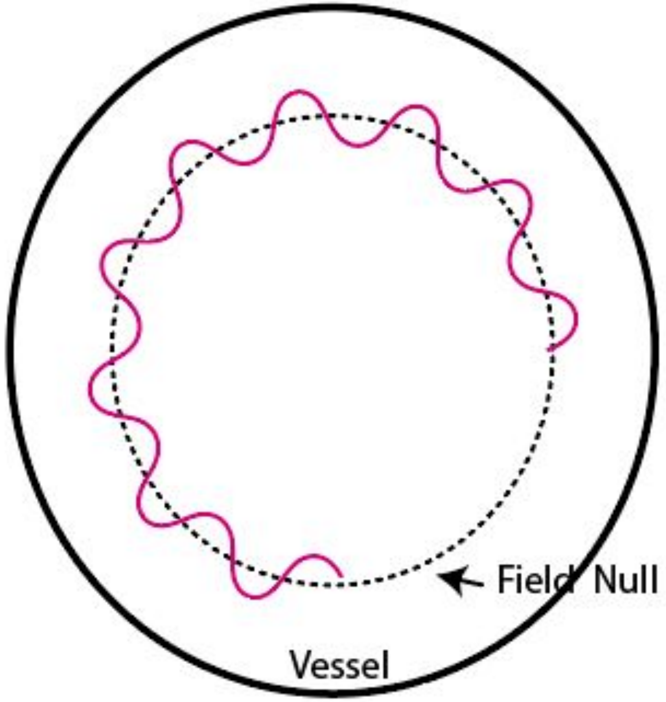
Runaway electron in ITER



An EP in the Radiation Belts

- Electrons & ions accelerated by instabilities
- Solar wind injects EPs into the magnetosphere

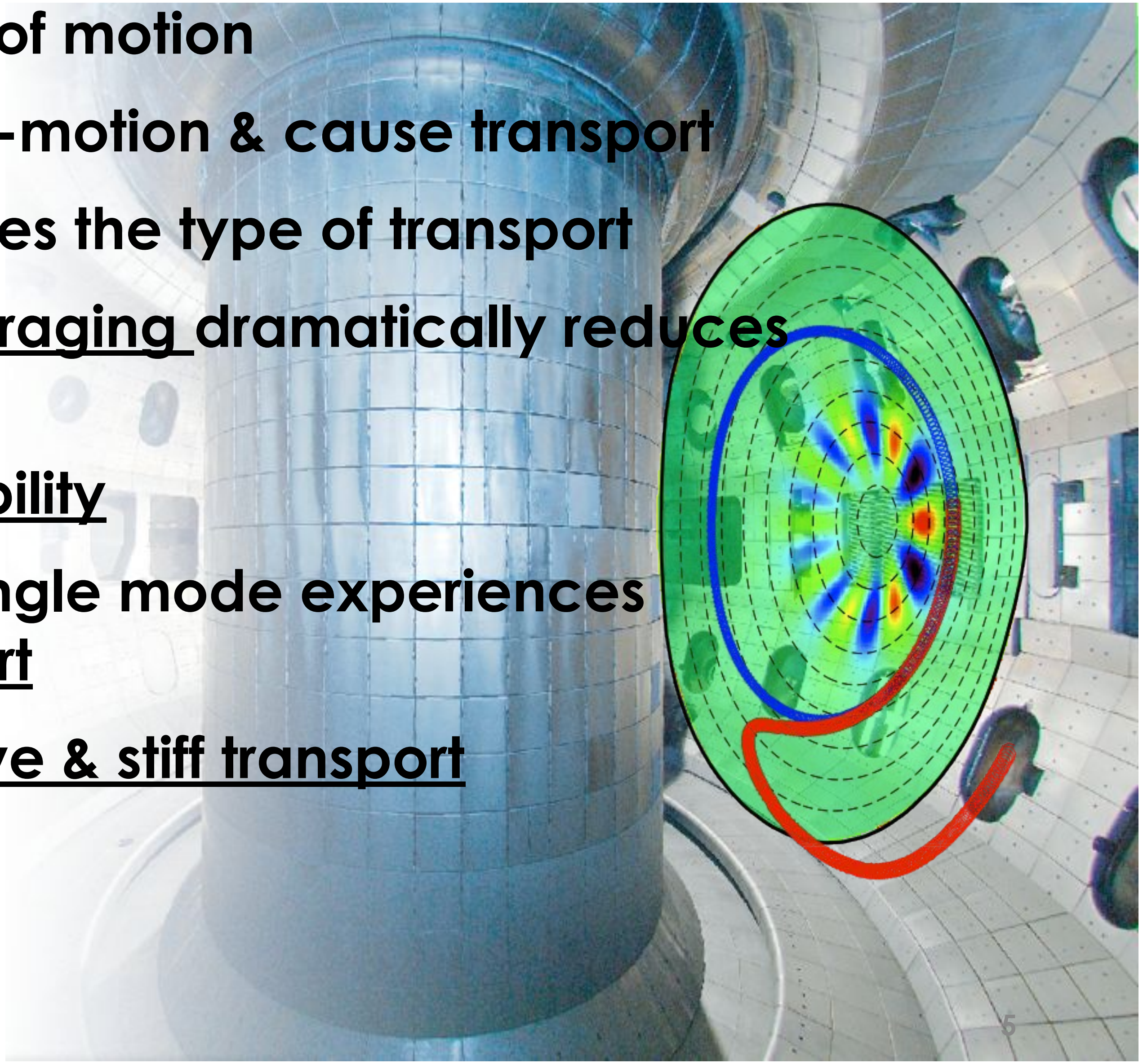
Beam ion in an FRC



- Neutral Beams
- Charged fusion products
- Acceleration by RF heating

Outline

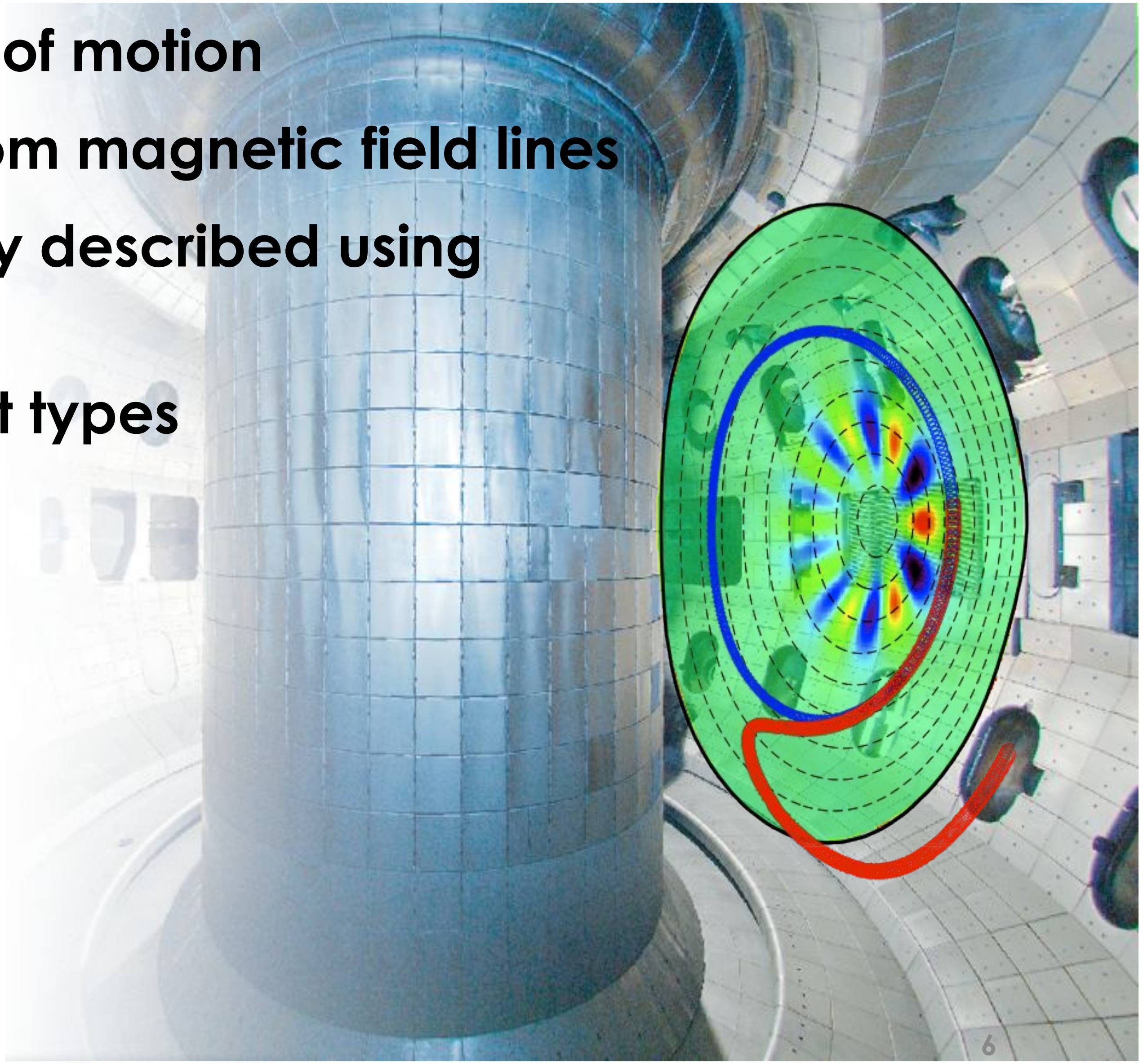
1. Orbits are described by constants of motion
2. Perturbations change constants-of-motion & cause transport
3. The wave-particle phase determines the type of transport
4. For non-resonant modes, orbit averaging dramatically reduces transport
5. Resonant particles can drive instability
6. An EP that stays in phase with a single mode experiences large convective resonant transport
7. Multiple resonances cause diffusive & stiff transport



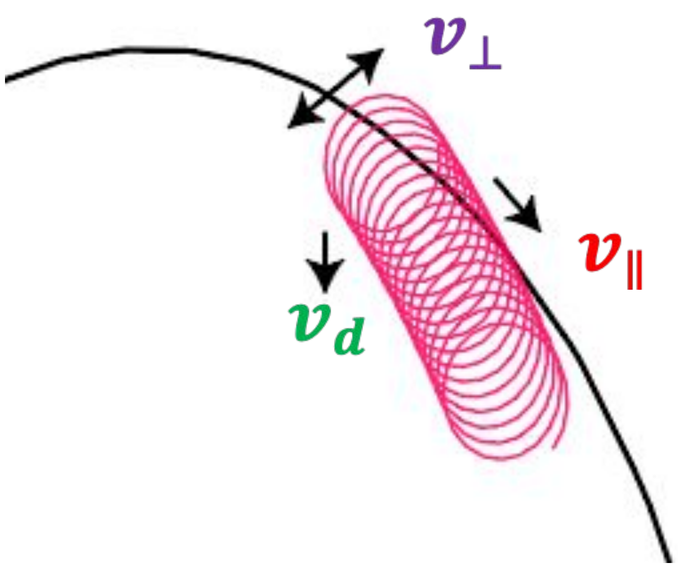
Outline

1. <u>Orbits</u> are described by constants of motion

- EP orbits have large deviations from magnetic field lines
- Complex EP orbits are most simply described using constants of motions
- Topological maps distinguish orbit types



EP orbits have large excursions from magnetic field lines



Flux
Surface
or
Field
Line

- Perp. Velocity $v_\perp$ → gyromotion
$\rho = v_\perp/\omega_c$

- Parallel velocity $v_\parallel$ → follows field line

- Curvature & Grad B drifts v_d → excursion from field line

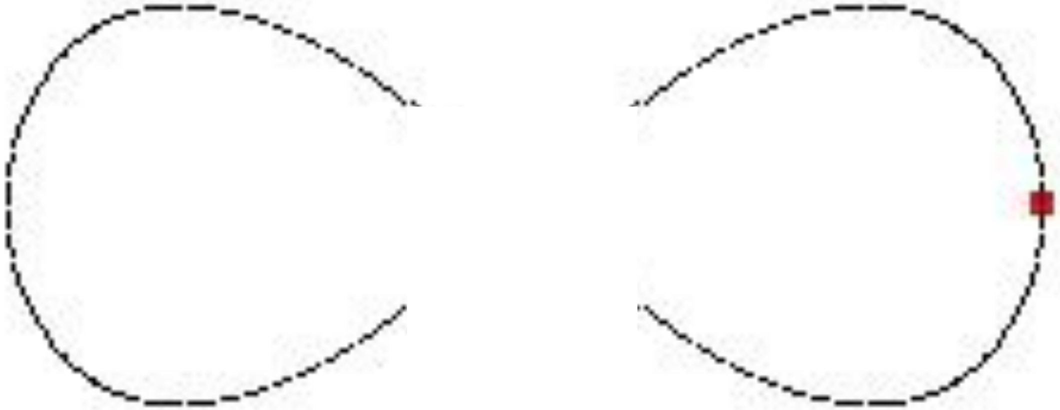
Parallel ~ v

Drift ~ $(v_\parallel^2 + v_\perp^2/2)$

□ Large excursions for large velocities

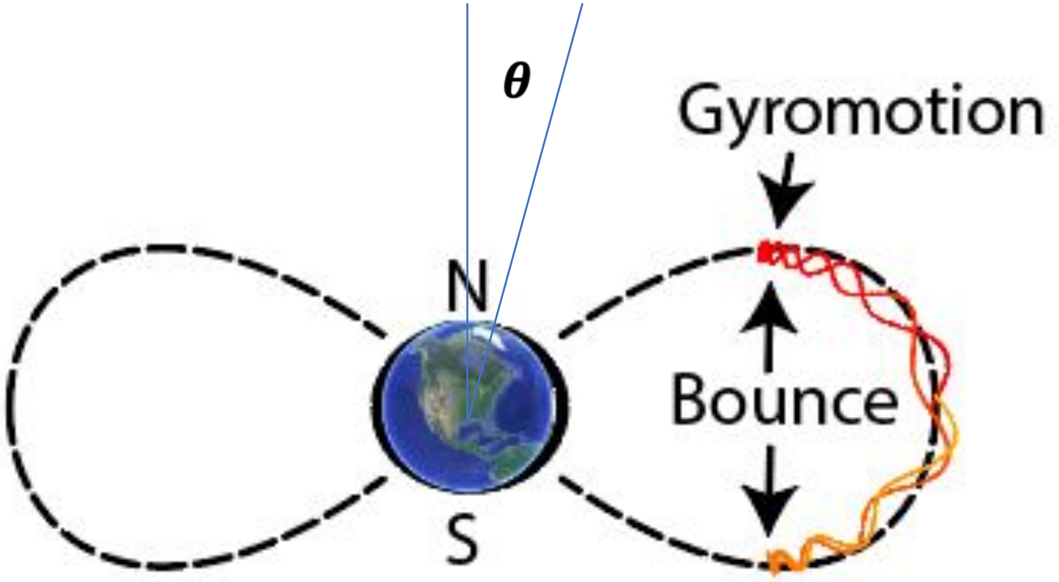
A dipole orbit has three periodicities: gyromotion, bounce & precession

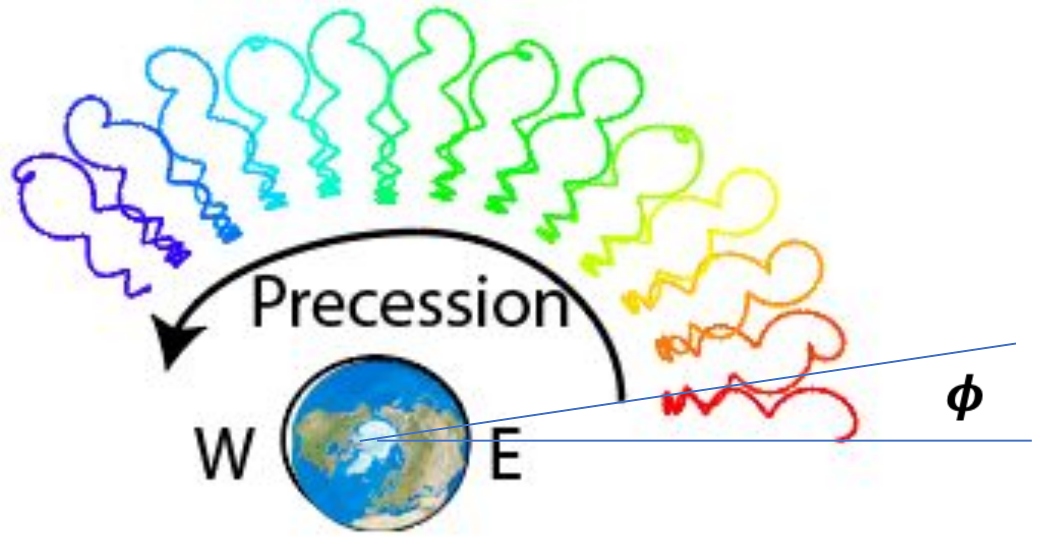
(R,z) Projection
Longitude



Top View
Latitude

A dipole orbit has three periodicities: ω_c, ω_θ, ω_ϕ

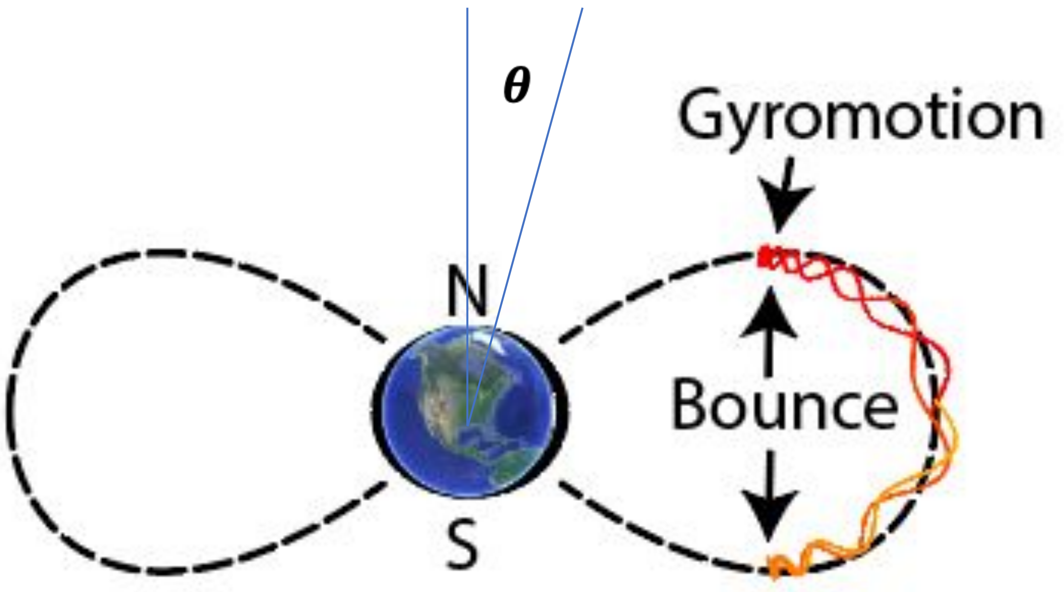


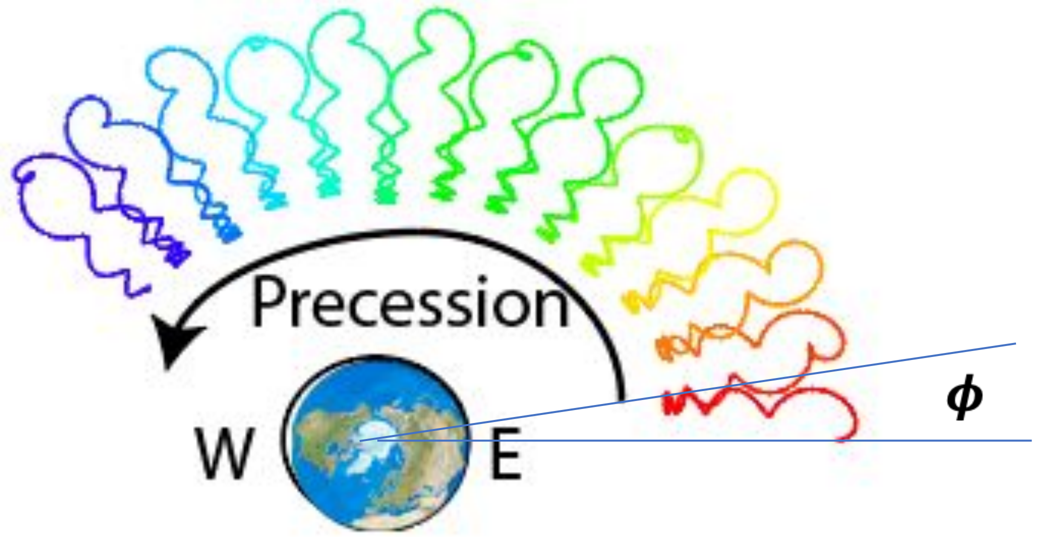


Two types of "constant-of-motion": Exact invariant & adiabatic invariant

- **Exact** associated with a symmetry--P_ϕ in an axisymmetric dipole field
- **Adiabatic** associated with an action, $J_i = \oint P_i dQ_i$, with Q_i, P_i, generalized coordinate & momenta

A dipole orbit has three periodicities: ω_c, ω_θ, ω_ϕ





Periodicity	Frequency	Adiabatic Invariant	
Gyromotion	ω_c	1st	$\mu = W_\perp / B$
Bounce	ω_θ	2nd	$J_\parallel = \oint v_\parallel ds$
Precession	ω_ϕ	3rd	$\propto \Psi$ (enclosed poloidal flux)

The use of constants-of-motion reduces the number of coordinates needed to describe an orbit

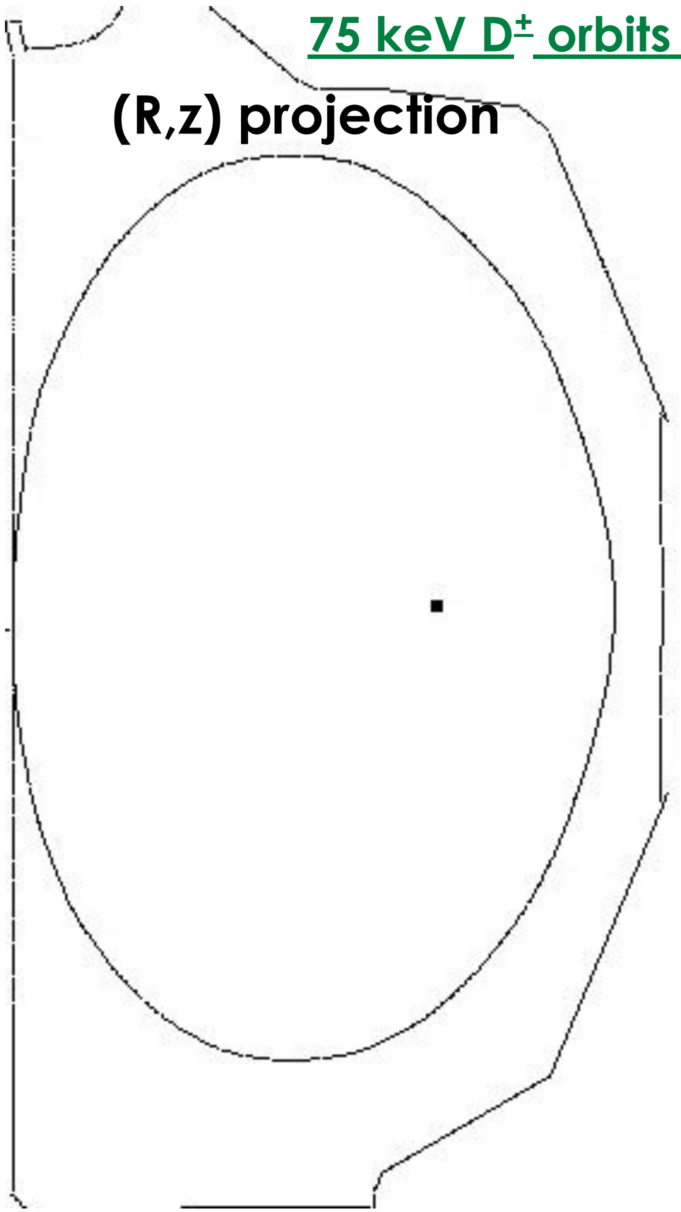
- 6 coordinates describe an arbitrary orbit in $(\vec{r}, \vec{v})$ space
- For example, if μ is conserved, don't need to follow the gyrophase to trace the orbit—we can follow the drift orbit rather than the full orbit.
- In an axisymmetric tokamak, only need 6-3=3 coordinates to describe an orbit because the energy W, the magnetic moment μ, and the toroidal canonical angular momentum P_ϕ are conserved.

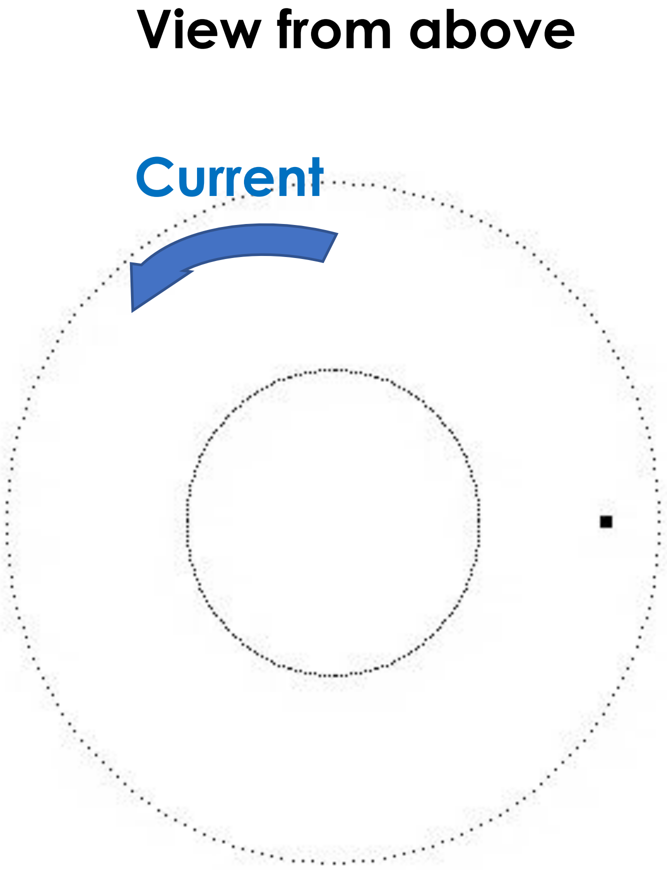
- Write* the distribution function as $F(W, \mu, P_\phi)$

*Note: Other coordinates can be more efficient in practice, e.g., (W, maximum radius of the orbit R_m, $v_\parallel / v$ at R_m) [Rome, Nucl. Fusion 19 (1979) 1193]

EPs with slightly different velocities can have very different orbits

<u>75 keV $D^{\pm}$ orbits in the DIII-D tokamak</u>

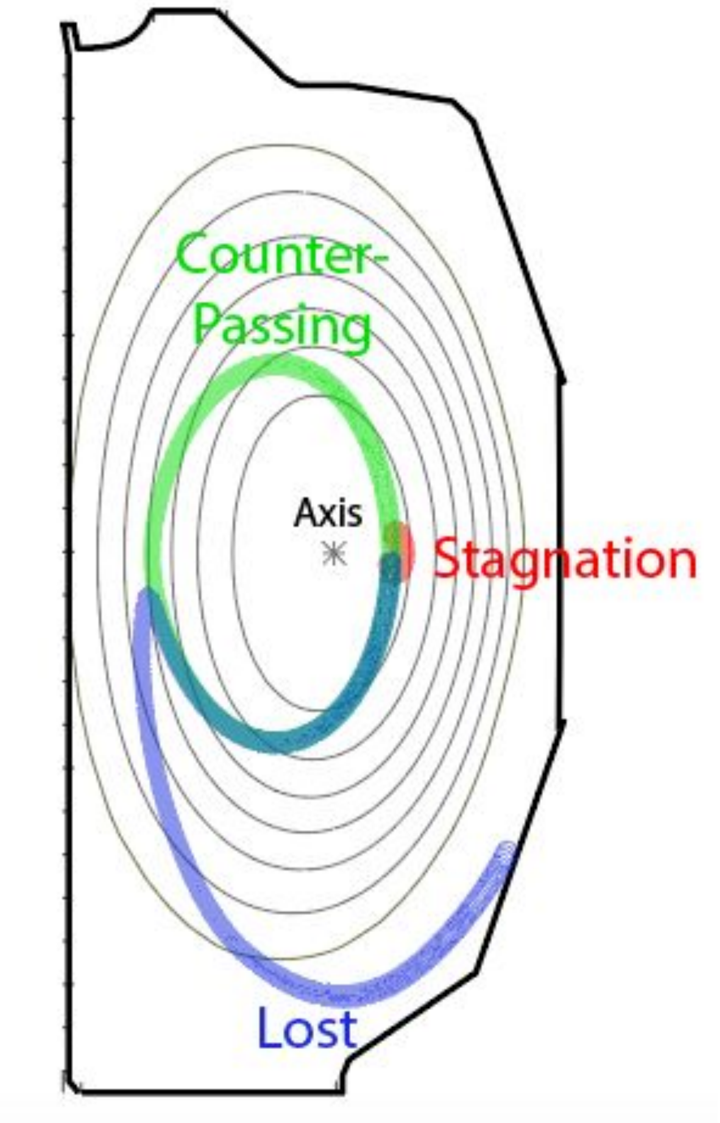


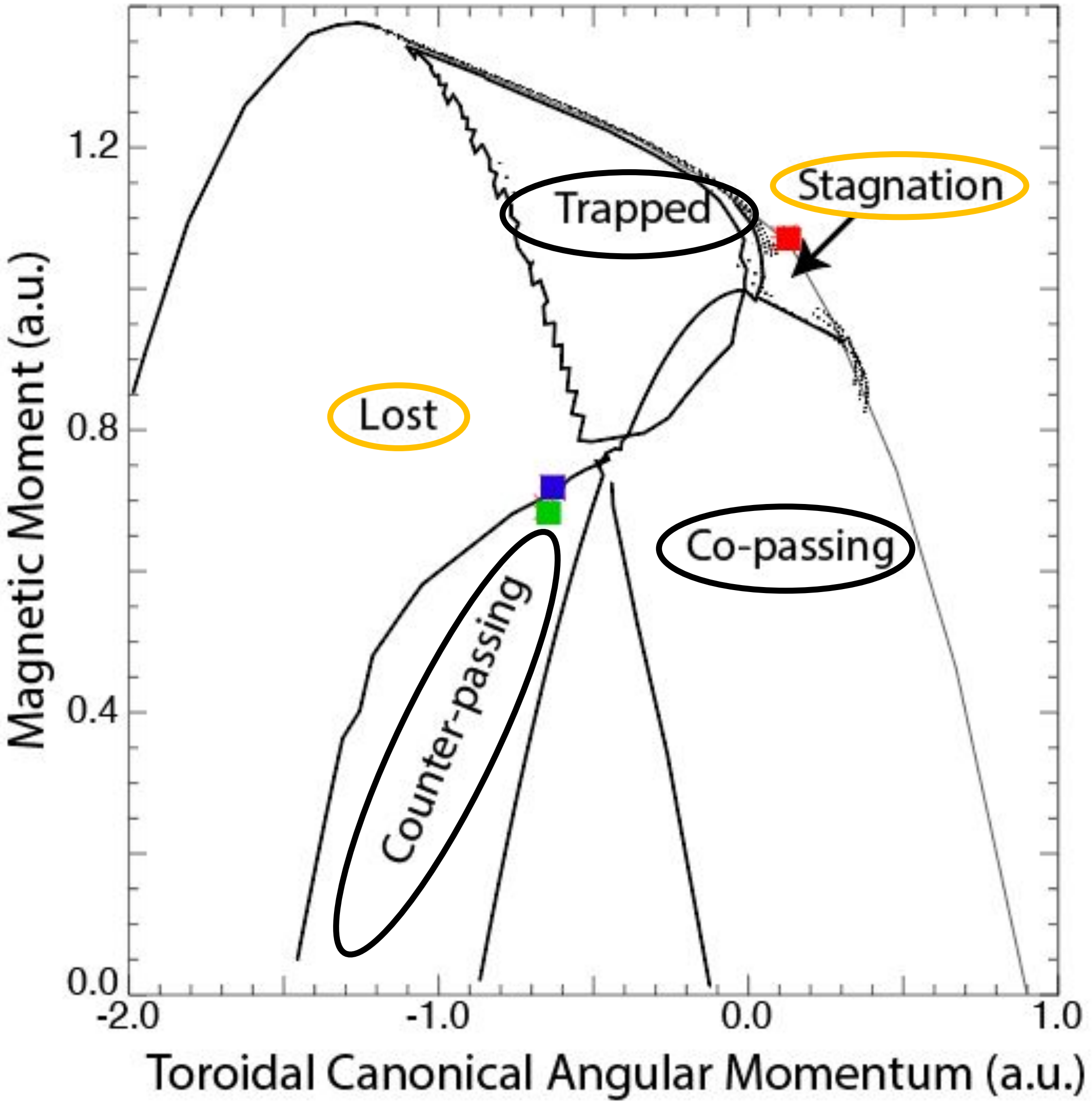


Terminology: For orbits, "co" and "counter" are relative to the plasma current (not B)

Complex EP orbits are most simply described using constants of motion

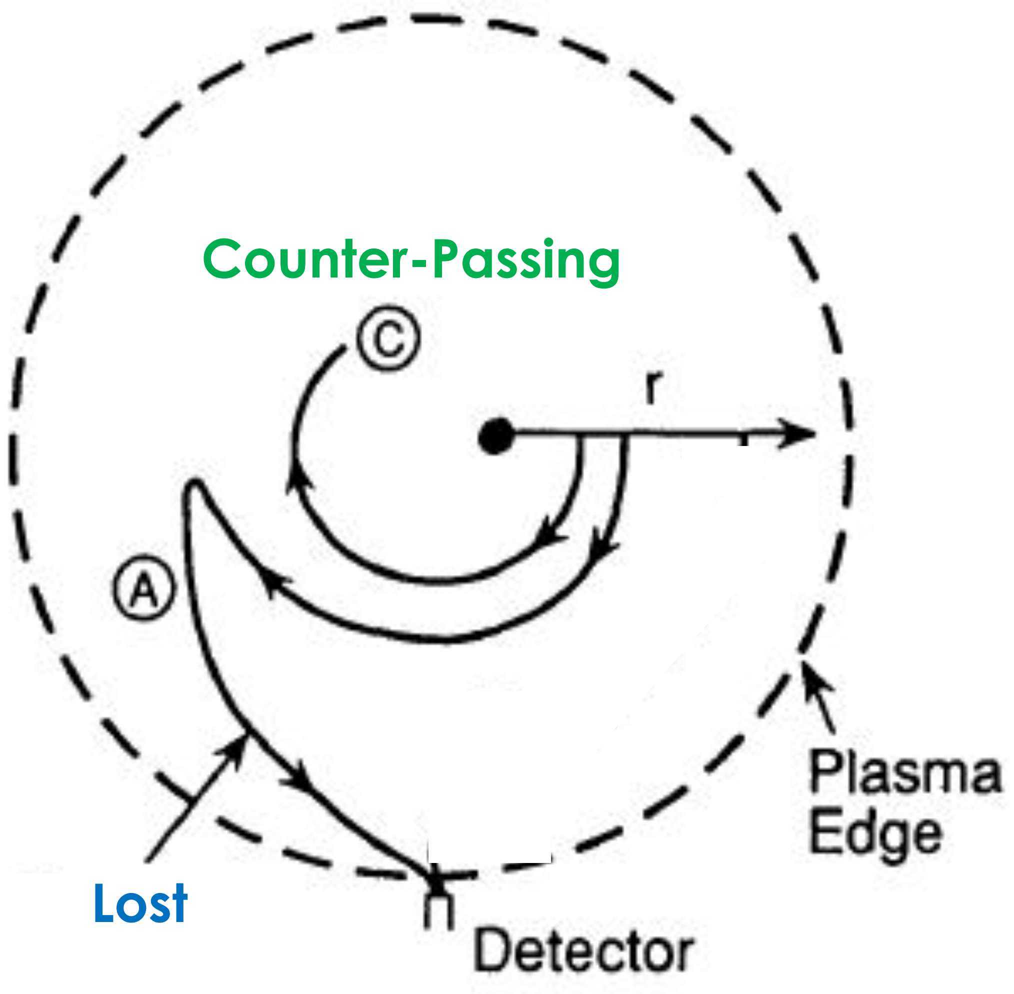
75 keV $D^{\pm}$ orbits in the DIII-D tokamak





An EP that crosses a topological boundary takes a LARGE transport step

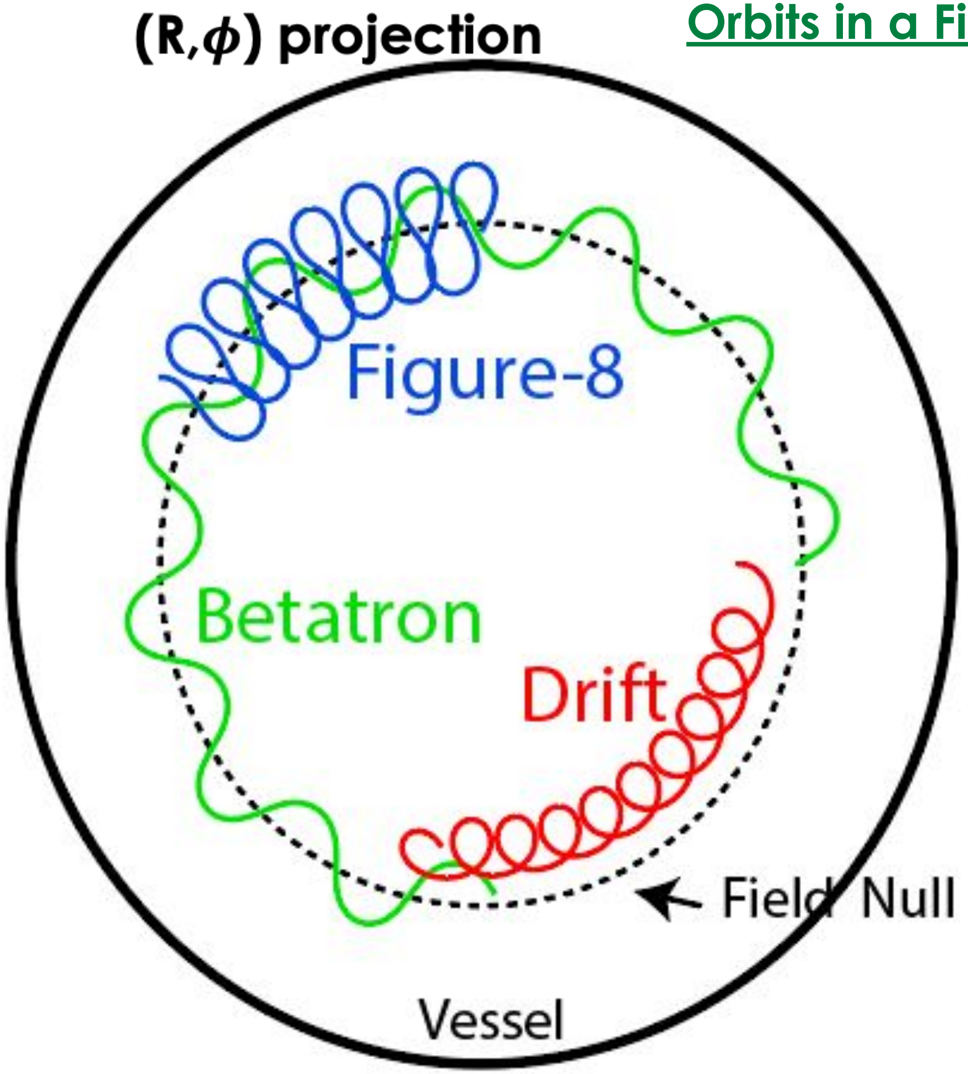
1 MeV triton orbits in the TFTR tokamak



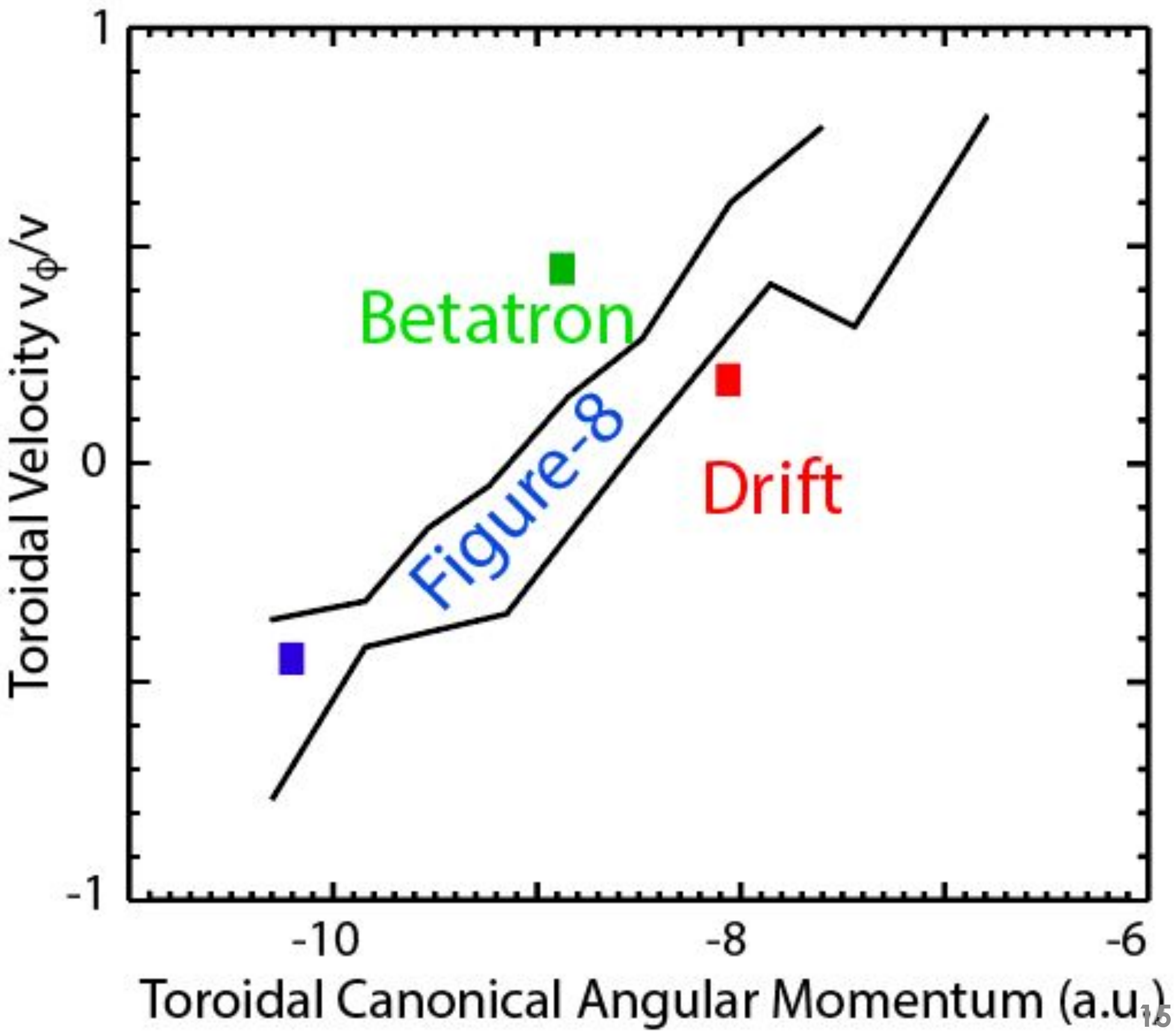
Zweben, Nucl. Fusion 31 (1991) 2219

- **Losses caused by fusion products moving from orbit C to orbit A were measured on TFTR (& many other tokamaks)**

Topological maps are useful for all configurations

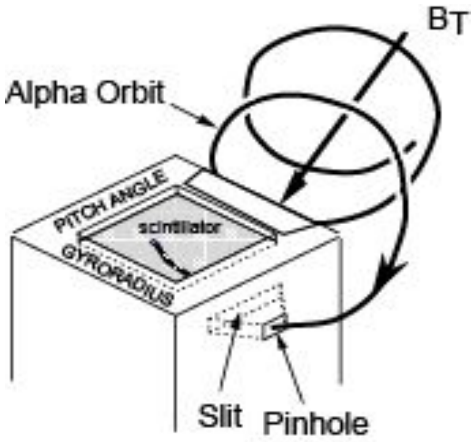


Orbits in a Field Reversed Configuration (FRC)

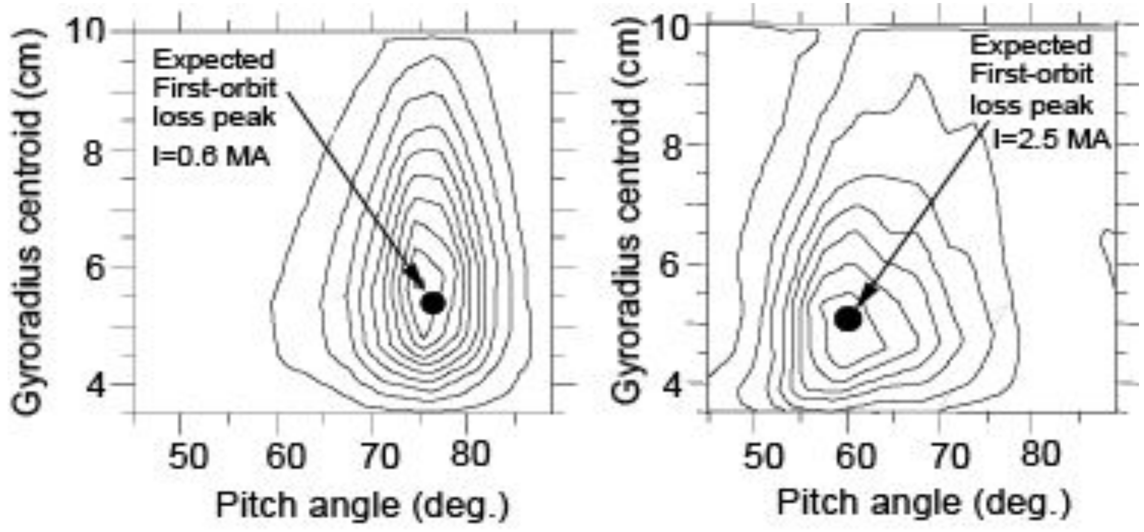


Axisymmetric orbit topology is well understood

Edge loss detector on the TFTR tokamak



Prompt losses of D-T alpha particles to a scintillator at the bottom of TFTR



Zweben, Nucl. Fusion 40 (2000) 91

Which quantity is conserved on orbital timescales for an EP in an axisymmetric tokamak?

1. Energy
2. Magnetic moment
3. Toroidal angular momentum, mRv_ϕ
4. Two of these
5. All of these

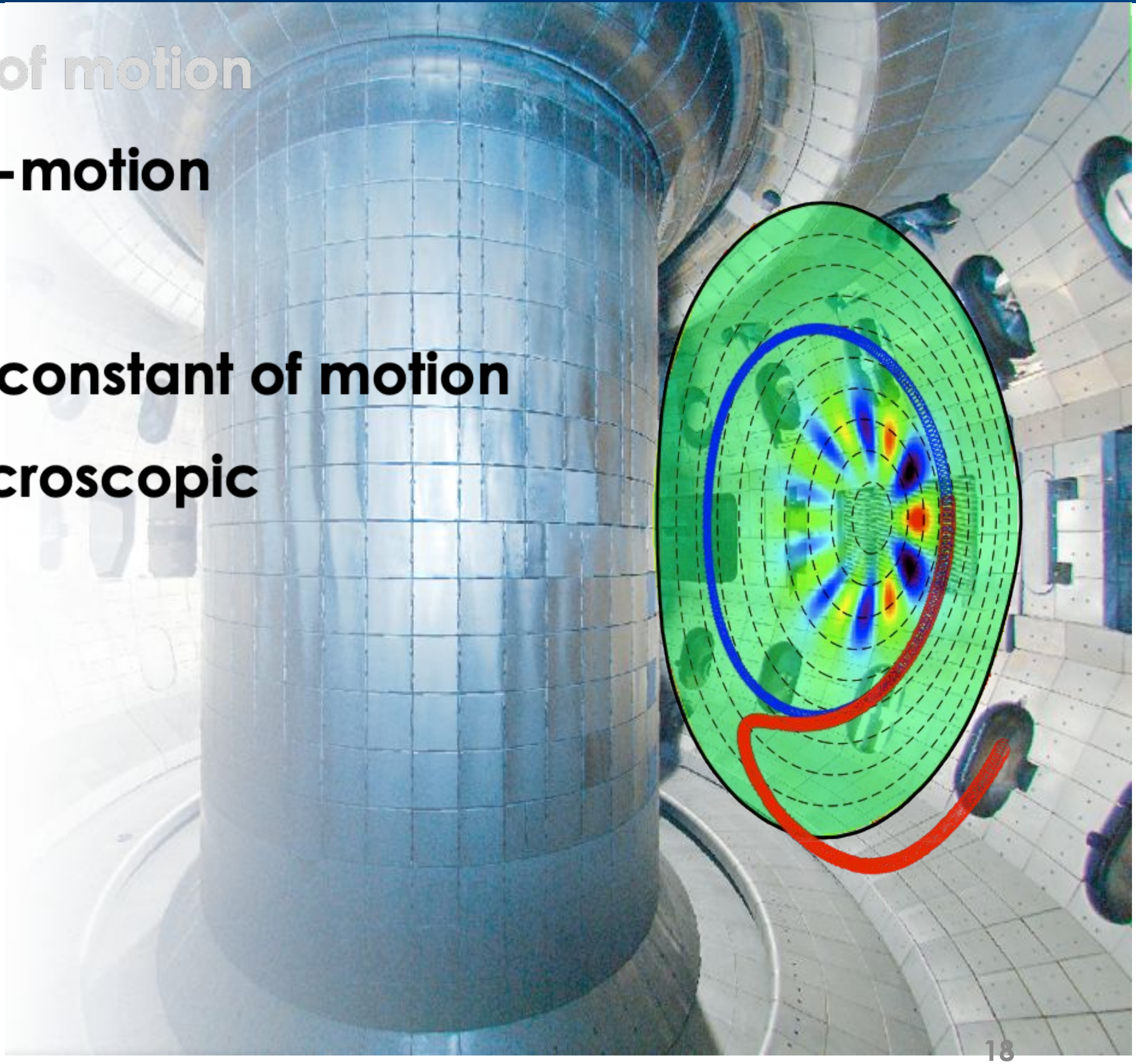
The toroidal canonical angular momentum is: $$P_\phi = mRv_\phi - Ze\Psi$$
($\Psi = RA_\phi$ is the poloidal flux—a radial coordinate)

Outline

1. Orbits are described by constants of motion

2. Perturbations change constants-of-motion

- Coulomb collisions
- General conditions for breaking a constant of motion
- Microscopic kicks $(\delta x, \delta t)$ and macroscopic irreversibility $(\Delta x, \Delta t)$



A single-particle picture is most appropriate for EPs

Property	Thermal	EP
Collisions	Strong	Rare
	↓	↓
Distribution function	Maxwellian	Complicated
	↓	↓
Transport theory	Fluid	Kinetic

There is a distinction between reversible and irreversible motion

- A buoy moves in a water wave but does not (necessarily) experience transport

There is a distinction between reversible and irreversible motion

- A buoy moves in a water wave but does not (necessarily) experience transport
- For transport, we take "snapshots" at intervals that are long compared to a wave period

$$\delta t \ll \Delta t \ll t$$

Microscopic reversible → Macroscopic irreversible → Fokker-Plank distribution function

Irreversible transport describes a group of EPs with similar initial conditions

Which is Forward; which is Backward?

Single Particle

Irreversible transport describes a group of EPs with similar initial conditions

Which is Forward; which is Backward?

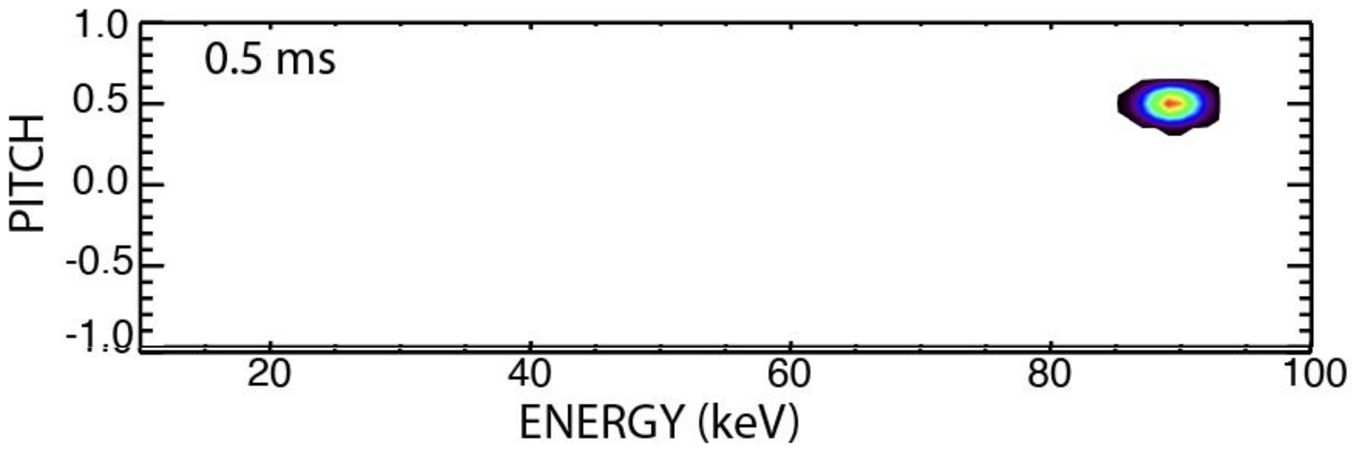
$$\langle \Delta x \rangle = 0, \langle (\Delta x)^2 \rangle > 0$$

Many Particles

- **Reversibility is disrupted by (a) multiple waves, (b) deterministic chaos, (c) collisions**

Collisions cause energy loss, pitch-angle scattering, and energy diffusion

- **A "blip" of fast ions start at $t = 0$ with the same energy & pitch ($v_{\parallel}/v$)**



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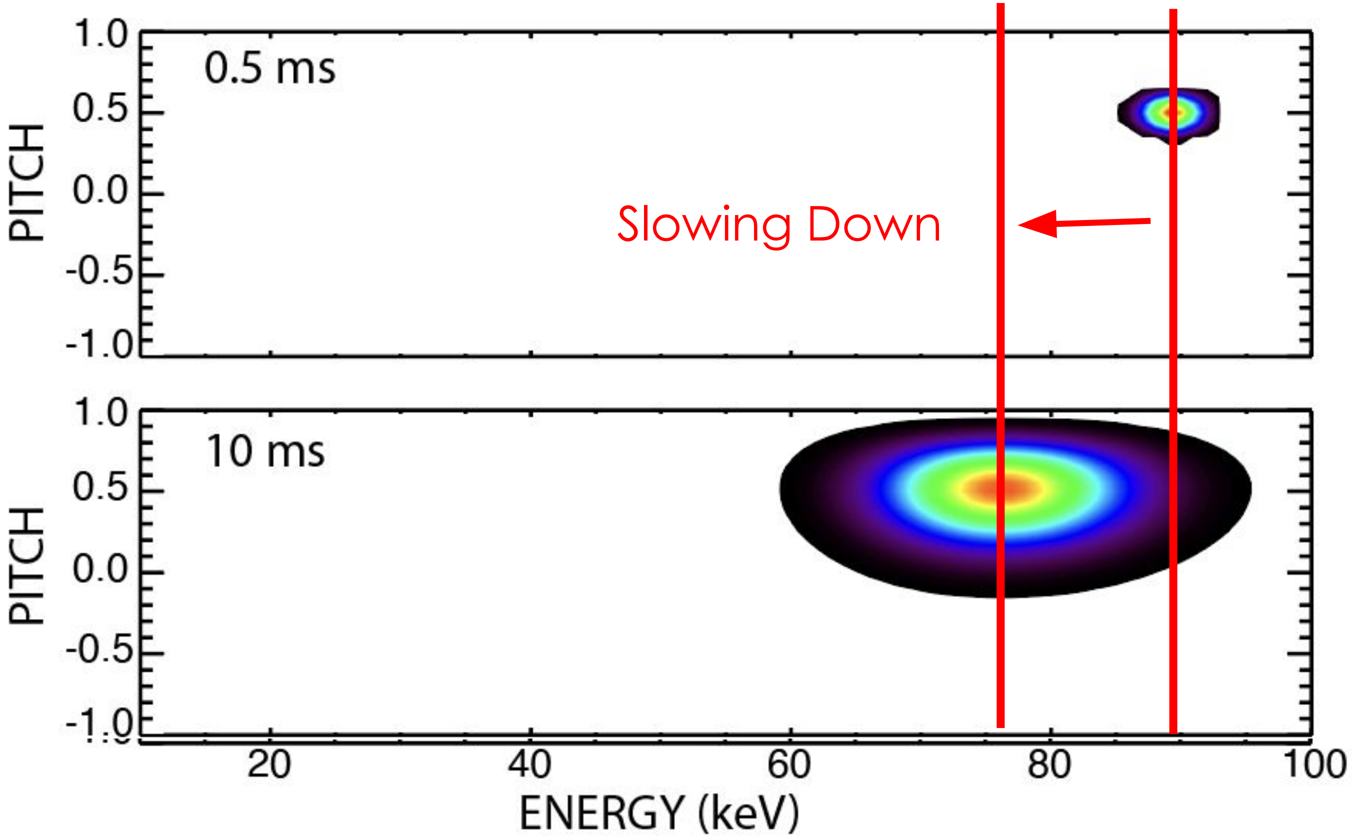

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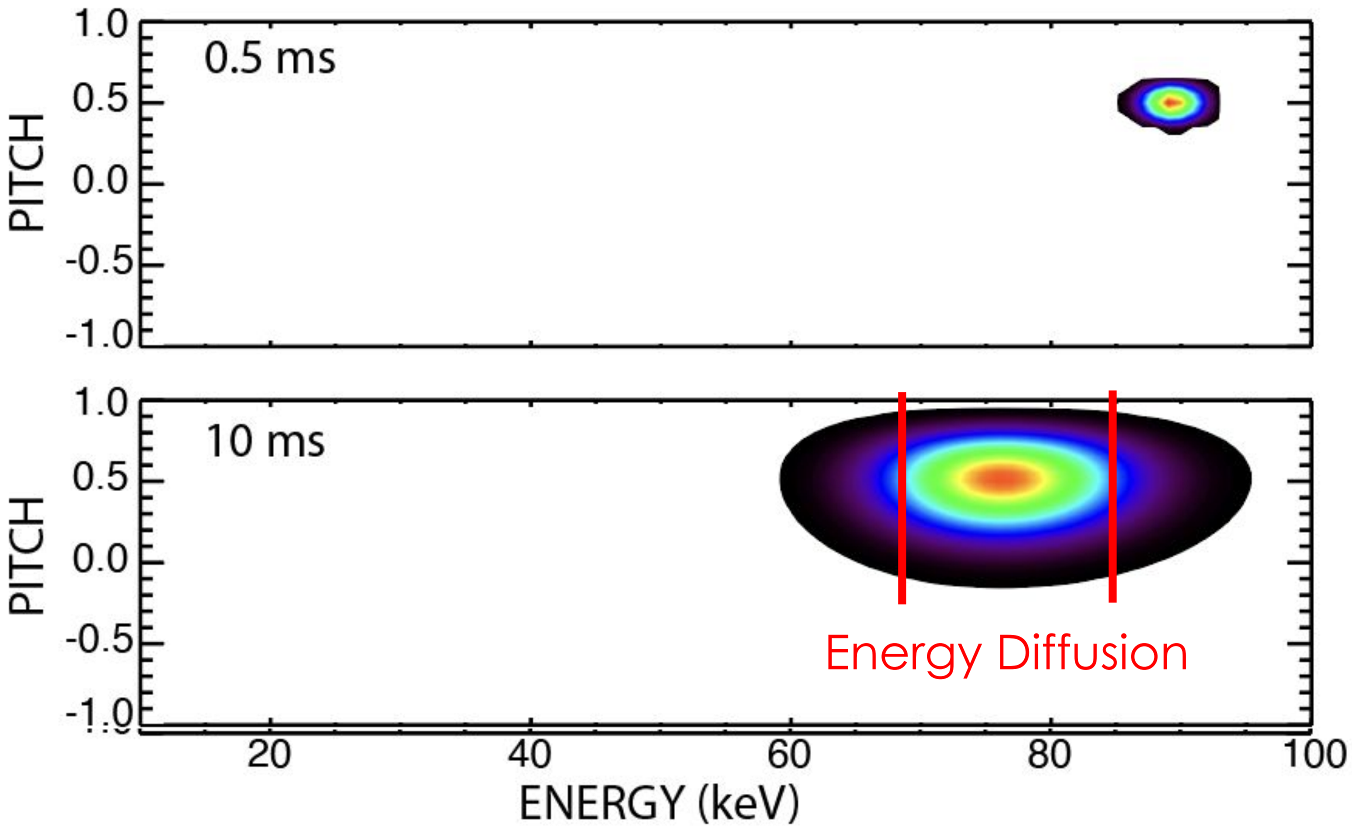

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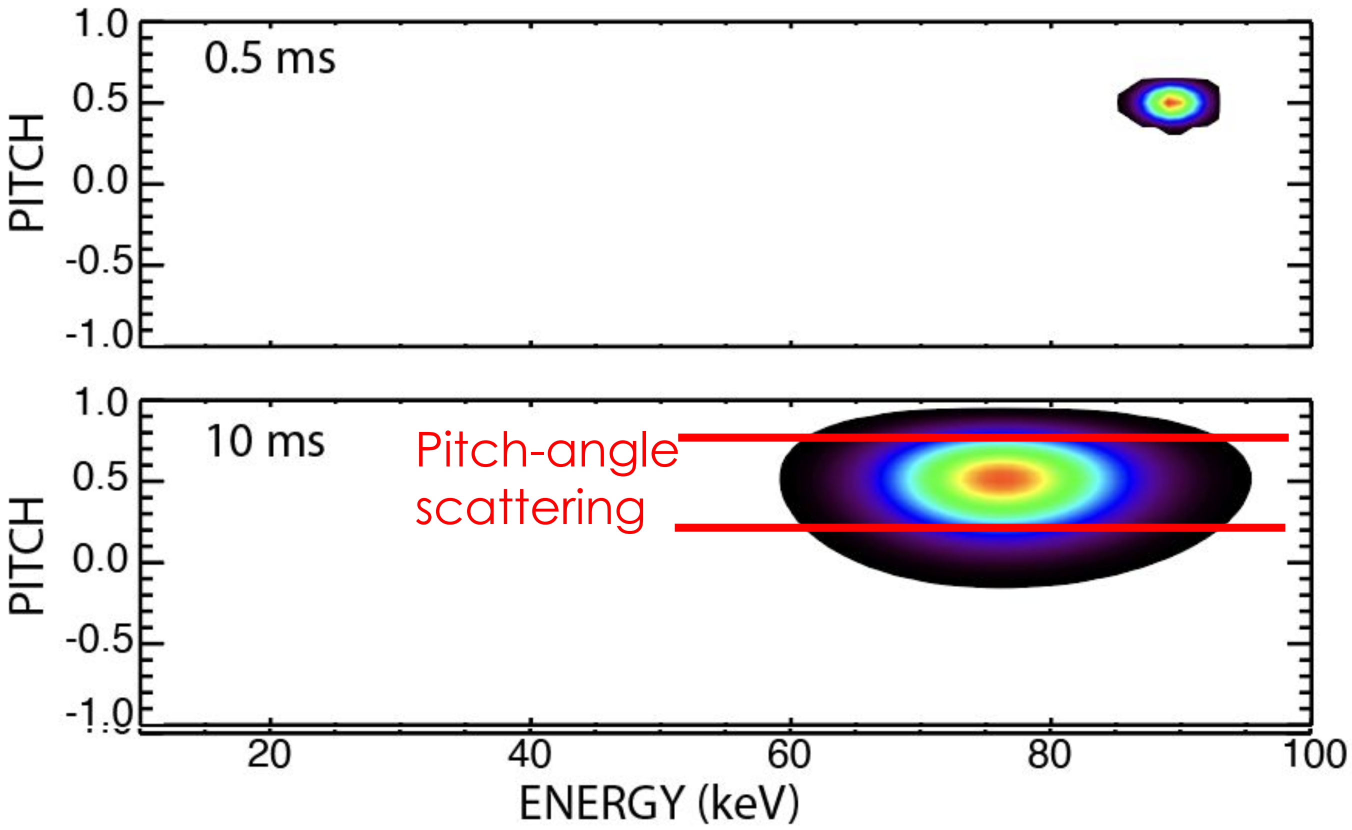

Collisions cause energy loss, pitch-angle scattering, and energy diffusion

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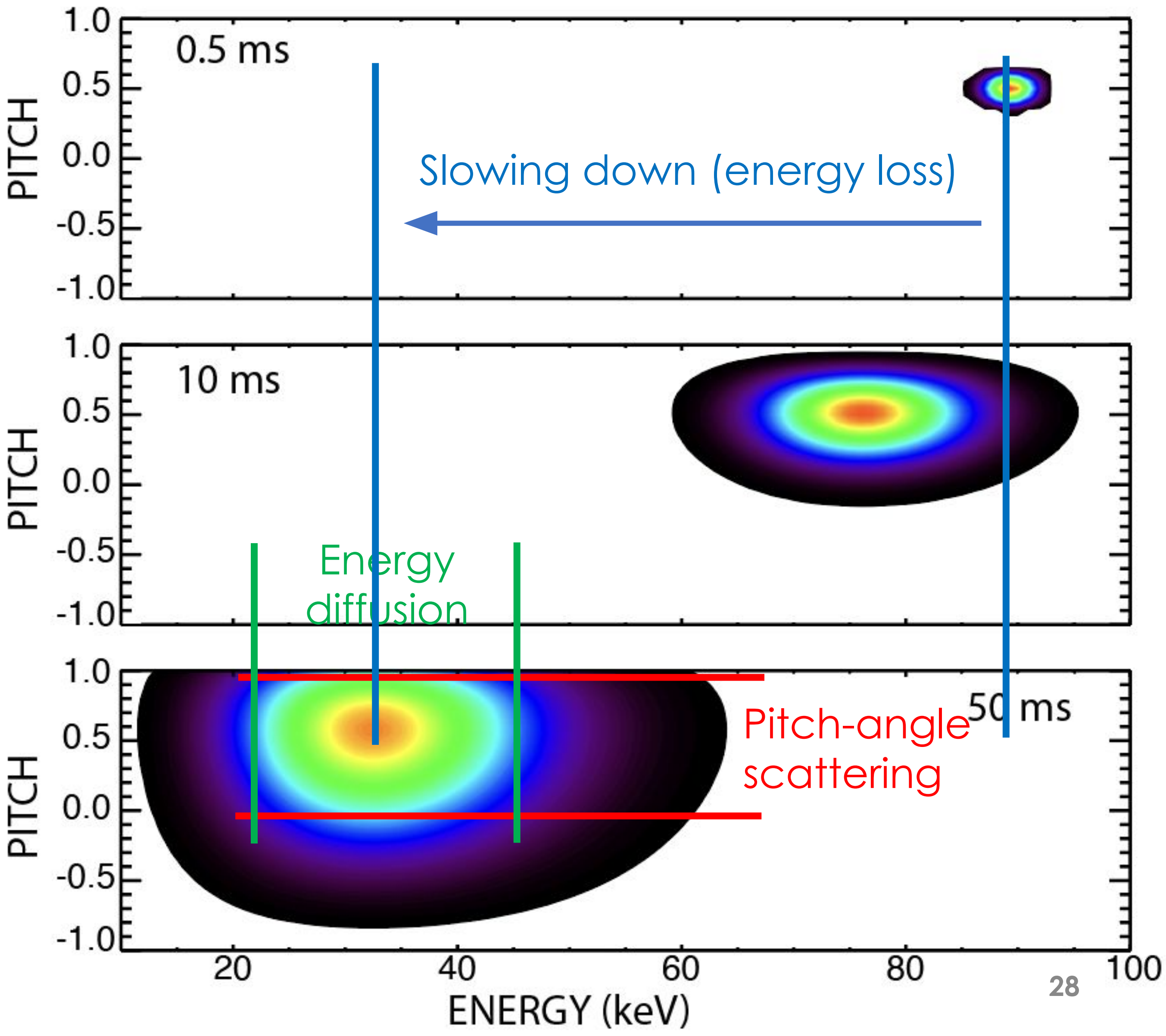


Fast ions slow down on electrons; slow down & pitch-angle scatter on thermal ions

Scattering rate $\nu \propto 1/v_{rel}^3$

$v_e \gg v_{EP} \gg v_i$

$n_e \sim n_i \gg n_f$

$\nu_{slow} \propto v_e^{-3} \propto T_e^{-3/2}$

$\nu_{slow}^e \gg \nu_{PAS}^e$

thermal electrons

fast ion

The energy where drag on electrons equals drag on ions is called the critical energy and is $\propto T_e$

fast ion

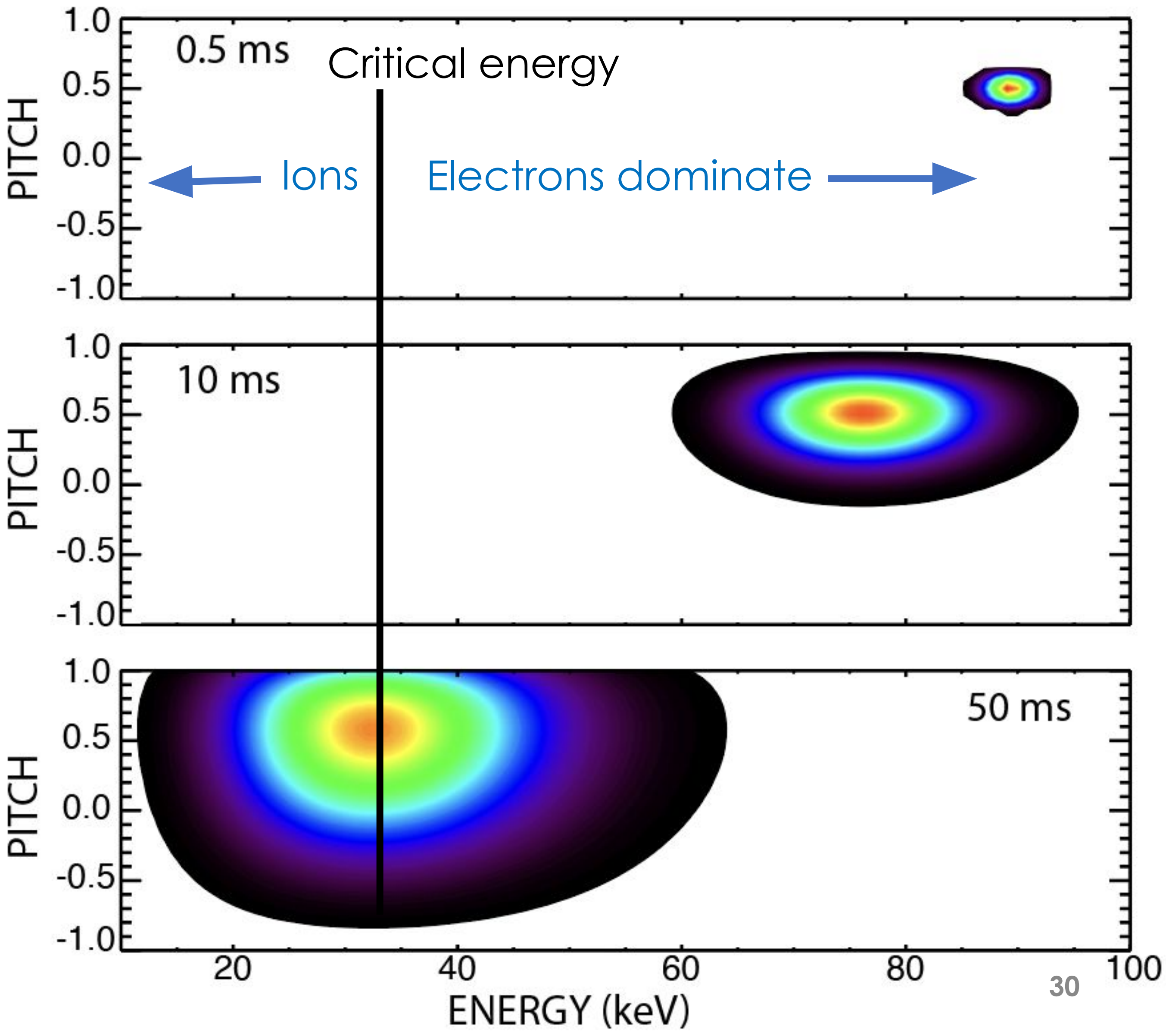
thermal ion

$\nu_{slow} \propto W^{-3/2}$

$\nu_{slow}^i \sim \nu_{PAS}^i$

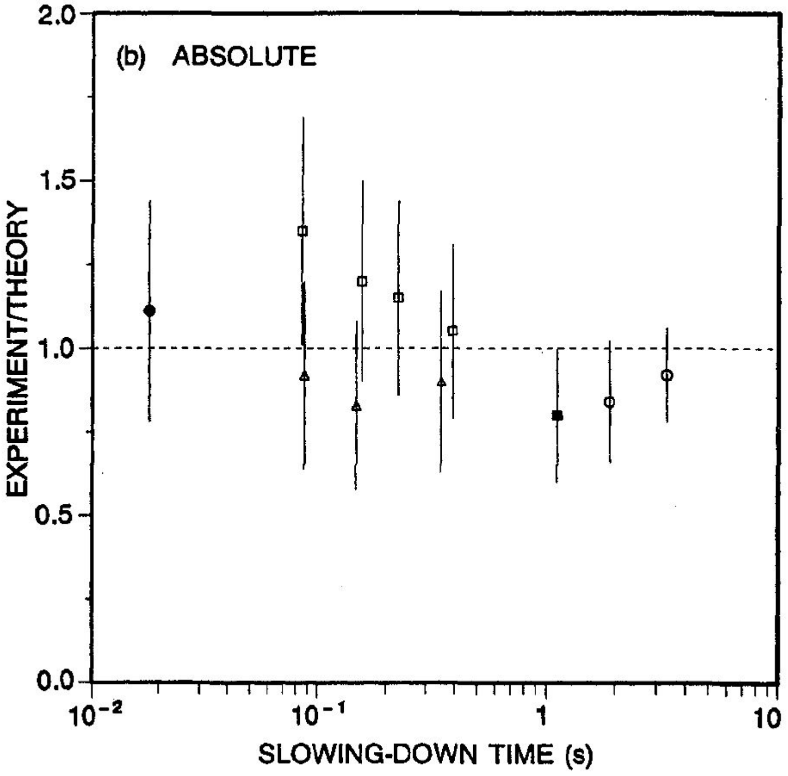
Pitch-angle scattering is more important at low fast-ion energies

The energy where drag on electrons equals drag on ions is called the <u>critical energy</u> and is $\propto T_e$



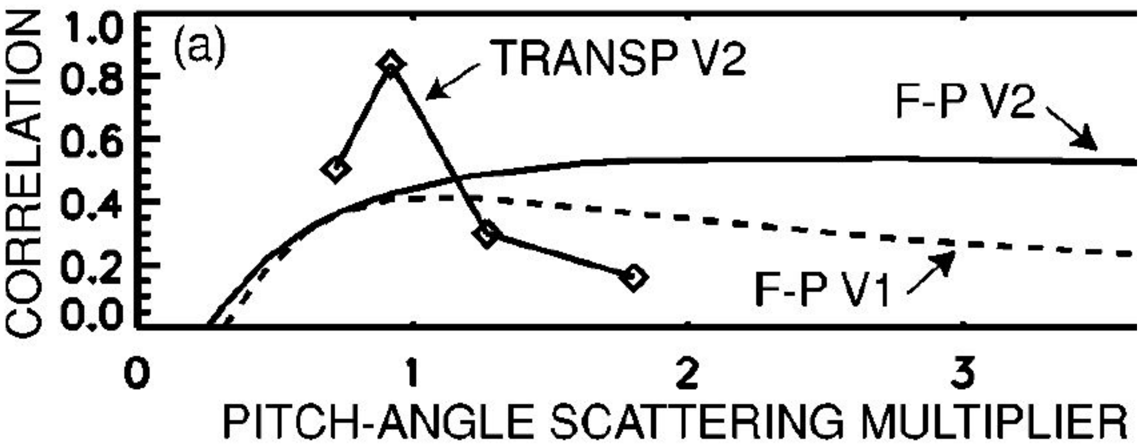
Fast-ion scattering rates are well validated experimentally

Slowing Down

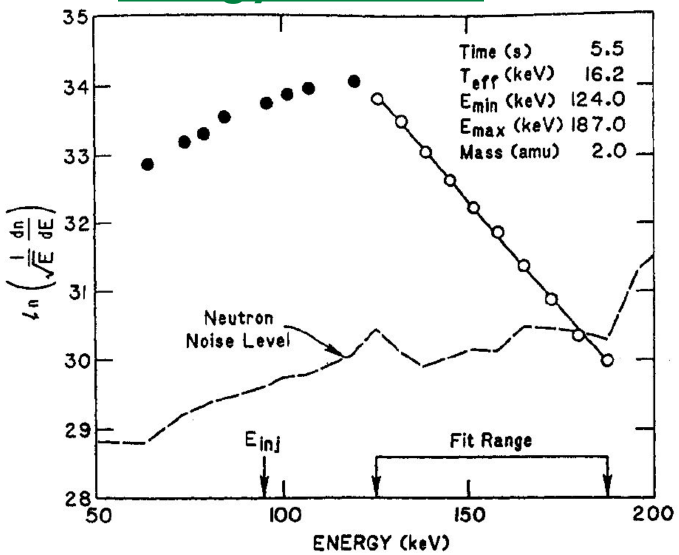


Heidbrink, Nucl. Fusion 34 (1994) 535

Pitch-angle Scattering



Energy Diffusion



Heidbrink, Phys. Pl. 9 (2002) 28

Fiore, Nucl. Fusion 28 (1988) 1315

Which scattering rate is larger for runaway electrons?

1. Slowing down
2. Pitch-angle scattering
3. About the same

Runaways slow down & pitch-angle scatter on thermal electrons & ions

Scattering rate $\nu \propto 1/v_{rel}^3$

$\boxed{v_{EP} \gg v_e, v_i}$

runaway

thermal electrons

thermal ions

There is a distinction between reversible and irreversible motion

ν_{PAS}

Coulomb deflection

$$\frac{\partial F}{\partial t} = C + S + Q$$

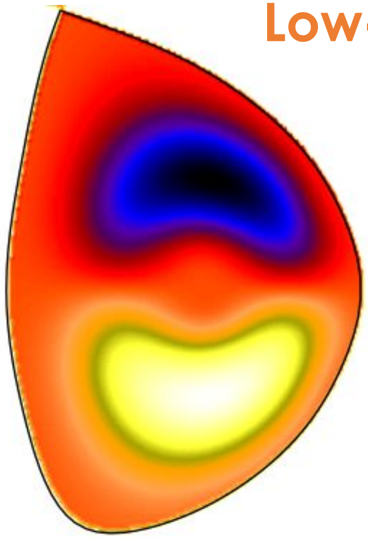
$$\delta t \ll \Delta t \ll t$$

Microscopic reversible → **Macroscopic irreversible** → **Fokker-Plank distribution function**

A wide range of perturbations can affect the energetic particles

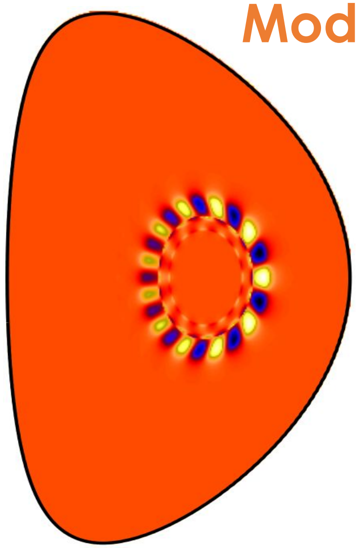
Decreasing Spatial Scale →

Low-n (0,1)



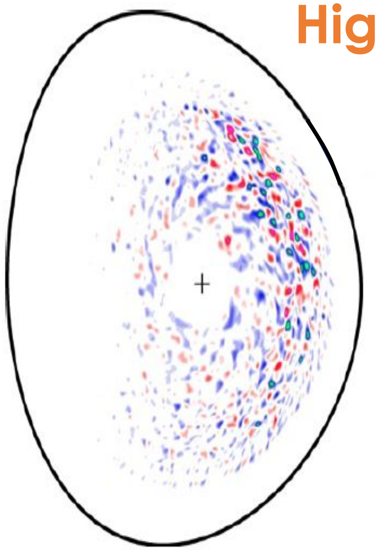
- Sawteeth
- Tearing Modes
- Off-axis Fishbone
- EGAM

Moderate-n



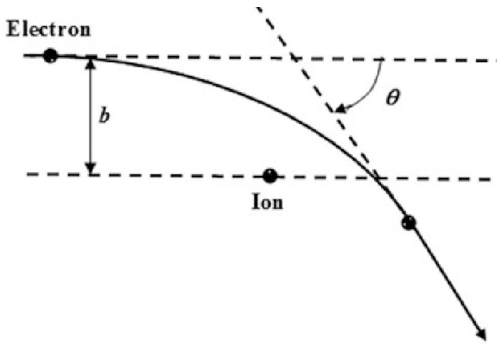
- Alfvén Eigenmodes
- Toroidal field ripple

High-n



Microinstabilities

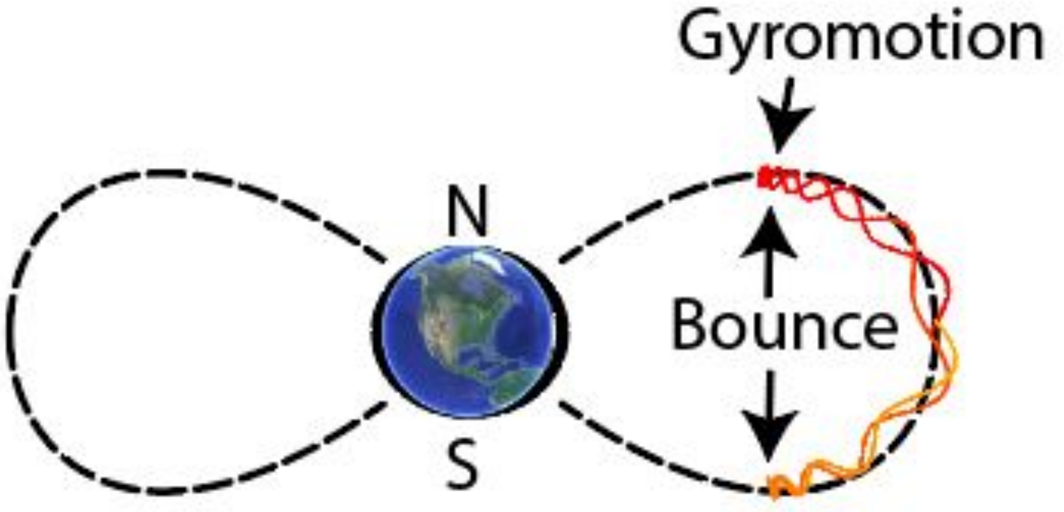
λ_D



Coulomb scattering

n = toroidal mode number
m = poloidal mode number

When invariants are conserved, confinement is good



- A "perturbation" is any $\vec{E}$ or $\vec{B}$ not included in the equilibrium.
- All frequencies including $\omega = 0$

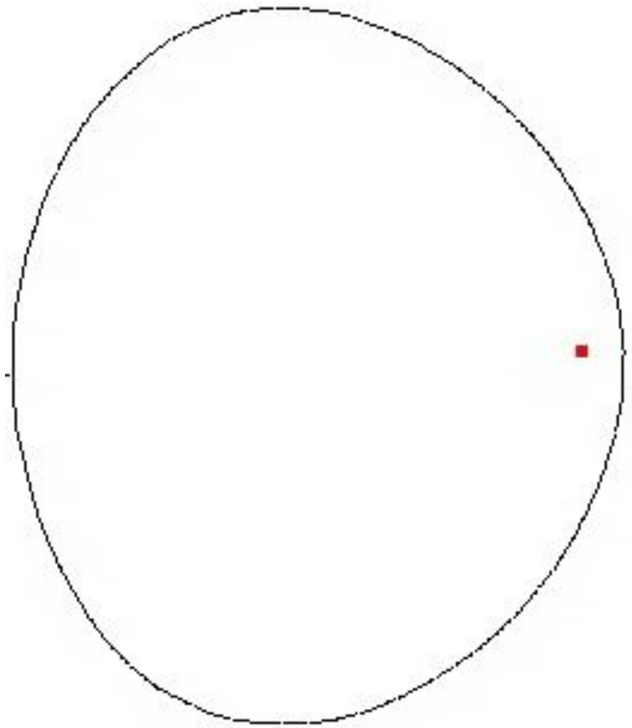
<u>Three ways to "break" an invariant</u>

1. EP sees "fast" changes, e.g., $\omega_c \delta t \sim 1$, $\frac{\varrho \nabla B}{B} \sim 1$ for μ
2. Resonant perturbation
3. Large perturbation

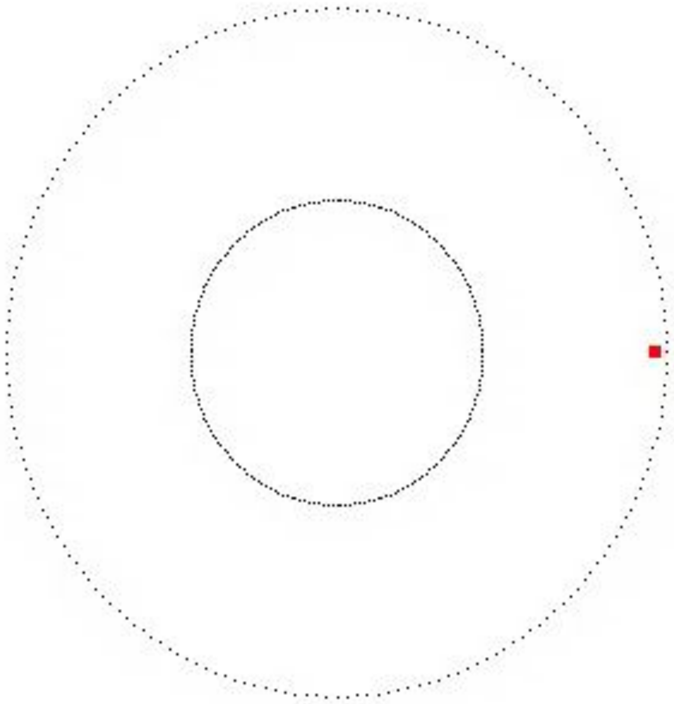
Breaking an invariant: Tokamak toroidal field ripple

- **Ideal tokamak is axisymmetric → P_ϕ is conserved**

(R,z) projection

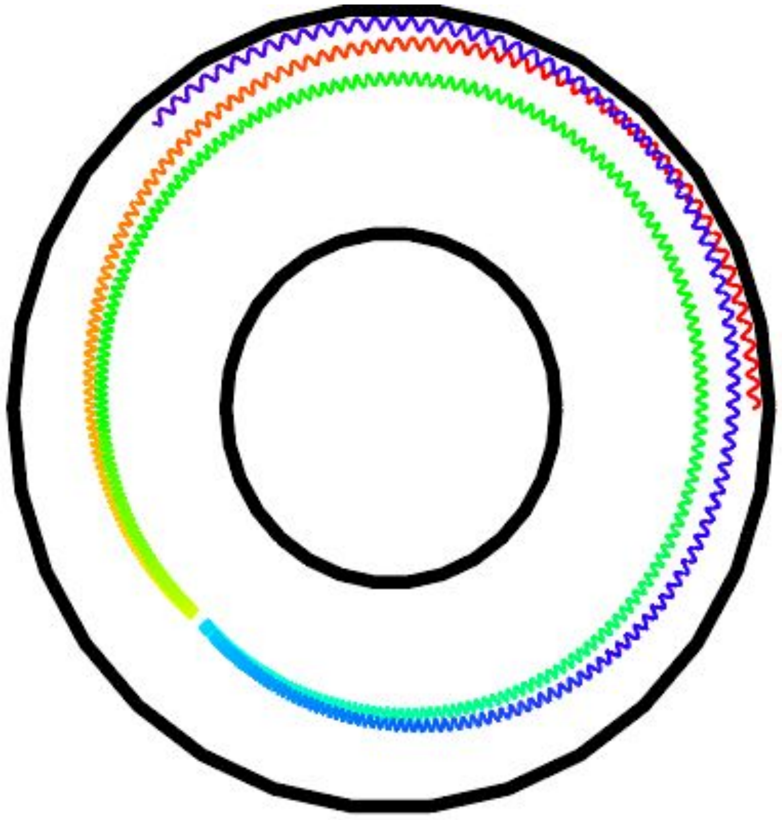


View from above



Breaking an invariant: Tokamak toroidal field ripple

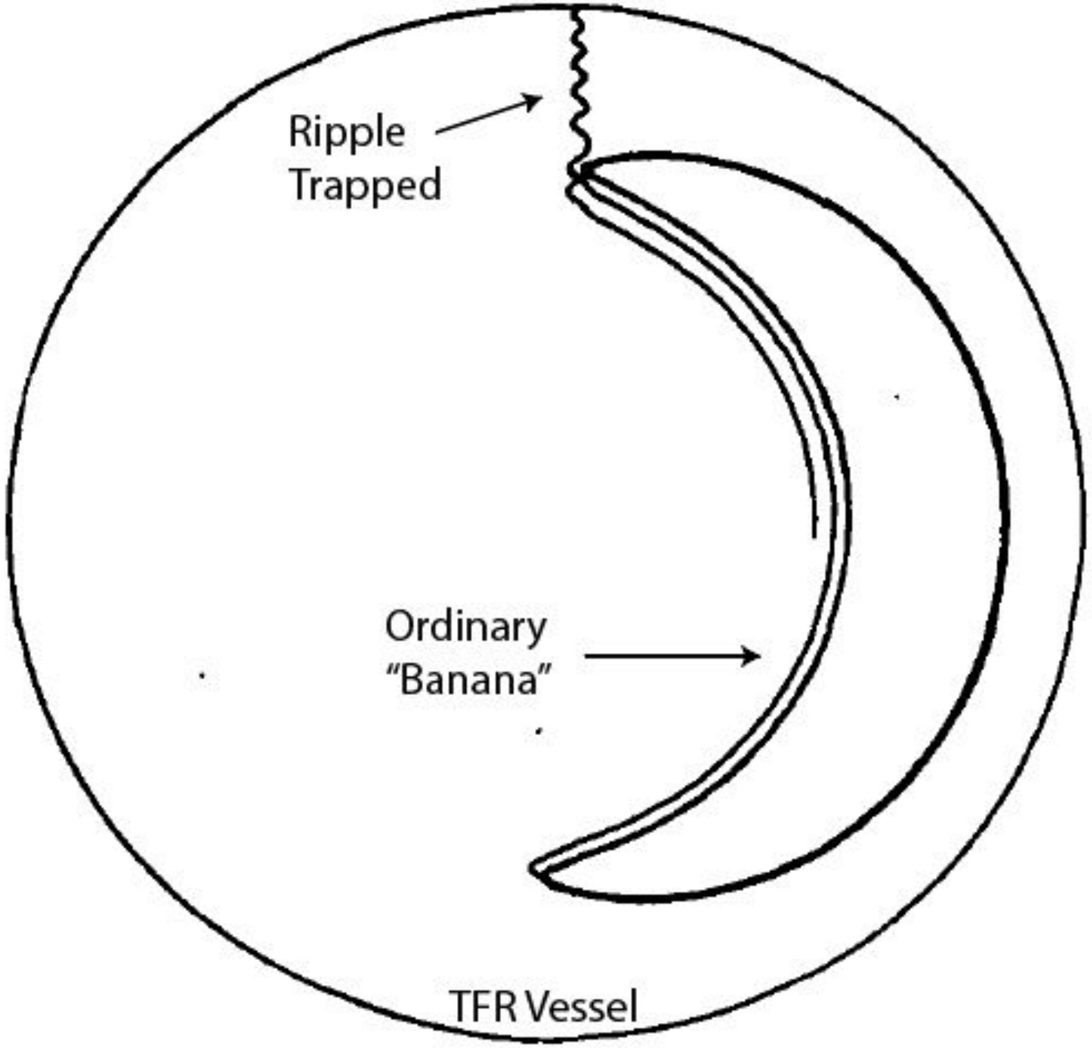
- **Ideal tokamak is axisymmetric → P_ϕ is conserved**
- **Real tokamak has field coils**
- **Large perturbations → broken P_ϕ**



Breaking an invariant: Tokamak toroidal field ripple

- Ideal tokamak is axisymmetric → P_ϕ is conserved
- Real tokamak has field coils
- Large perturbations → broken P_ϕ
- Experiments on ISX-B[1] and JET[2] halved the # of field coils → huge reduction in trapped fast ions
- Elimination of these "super-bananas" a key aspect of stellarator design[3]

Calculated orbit in the TFR tokamak



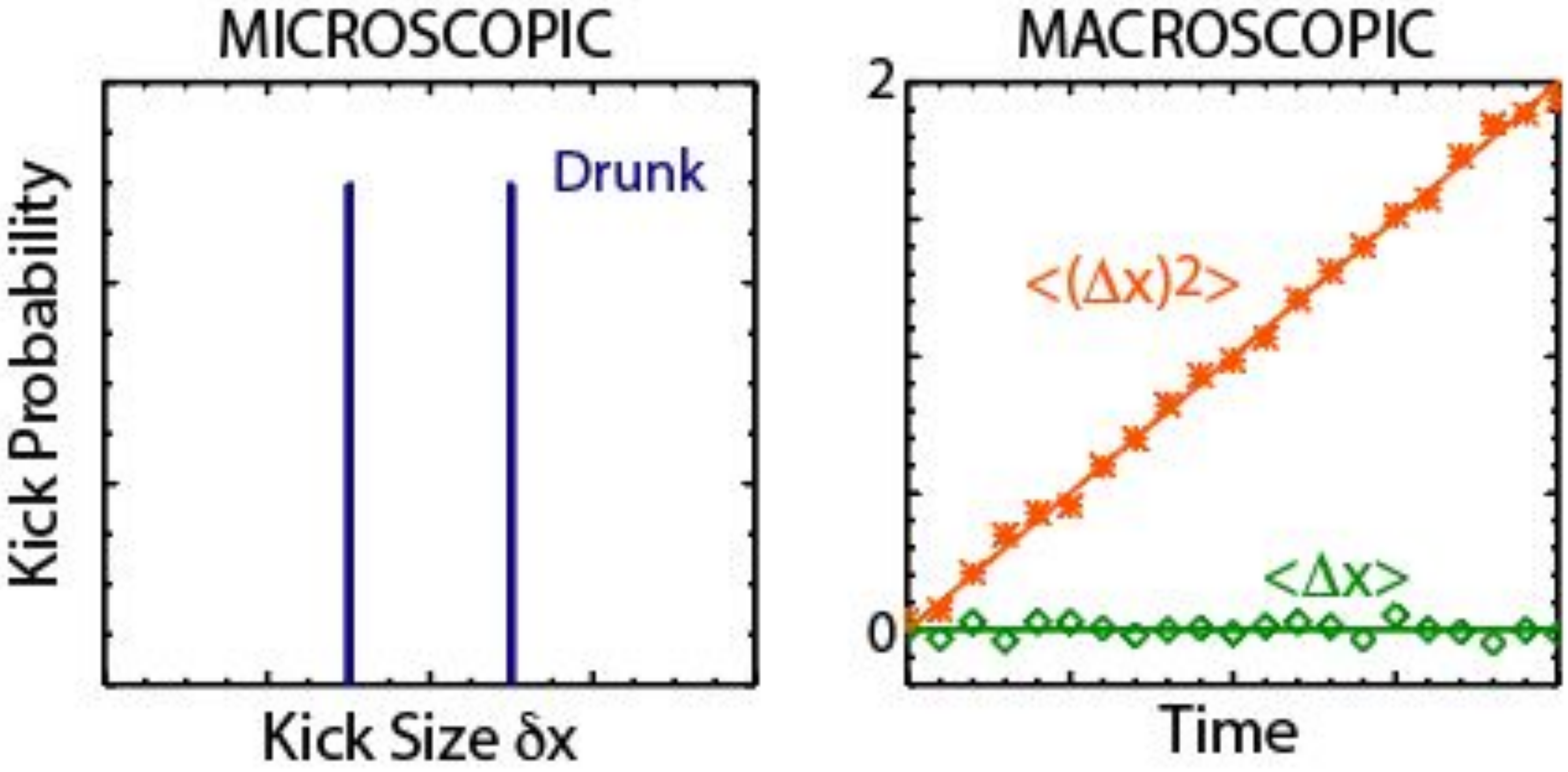
Goldston, J. Plasma Phys. 26 (1981) 283

[1]Scott, Nucl. Fusion 25 (1985) 359

[2]Sadler, Pl. Phys. Cont. Fusion 34 (1992) 1971

[3]Mynick, Phys. Pl. 13 (2006) 058102

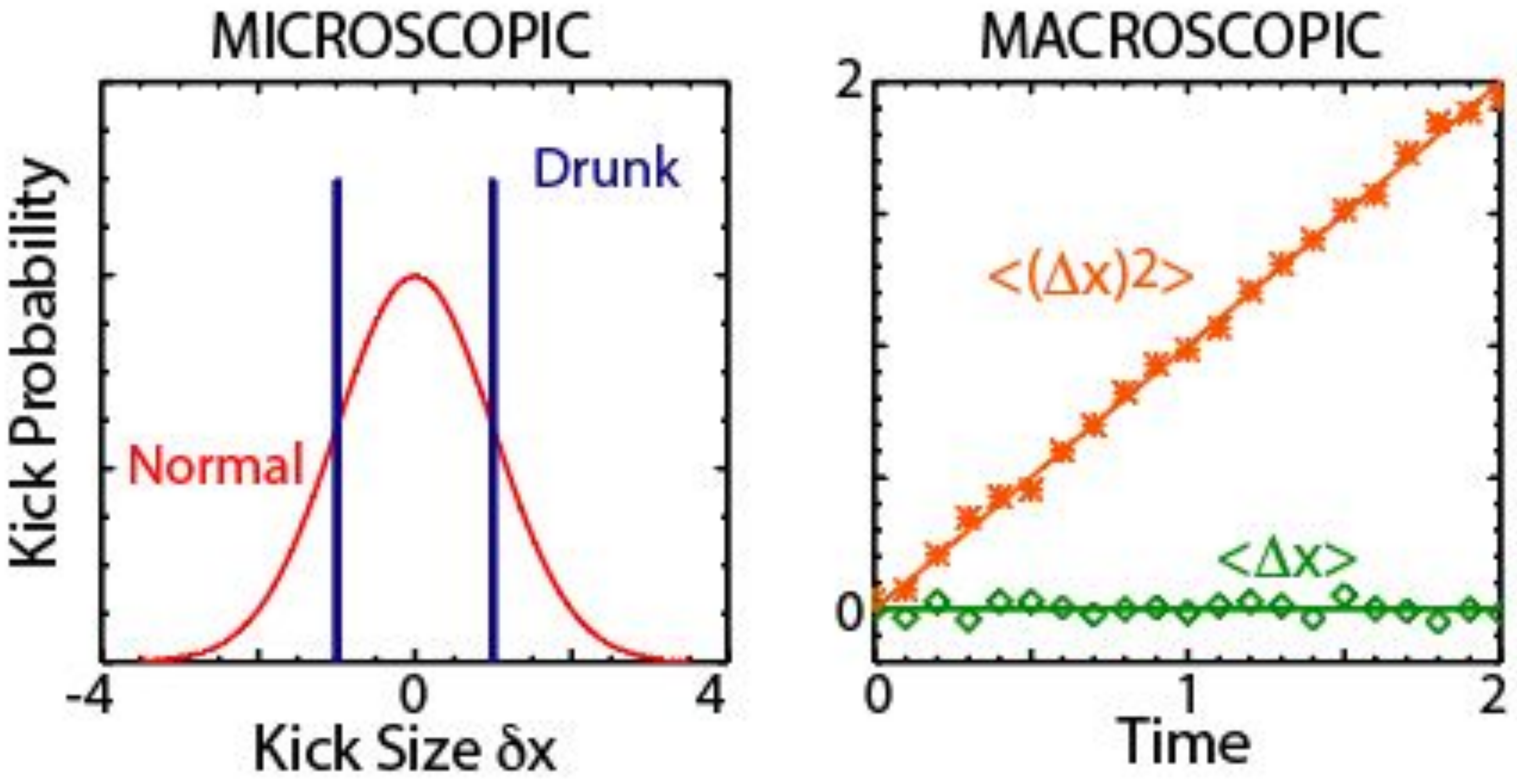
The microscopic kicks determine the macroscopic behavior



Random Walk: $D \approx (\delta x)^2/(2\delta t)$

Diffusion: $\langle(\Delta x)^2\rangle = 2Dt^\gamma$ **with** $\gamma = 1$

The microscopic kicks determine the macroscopic behavior



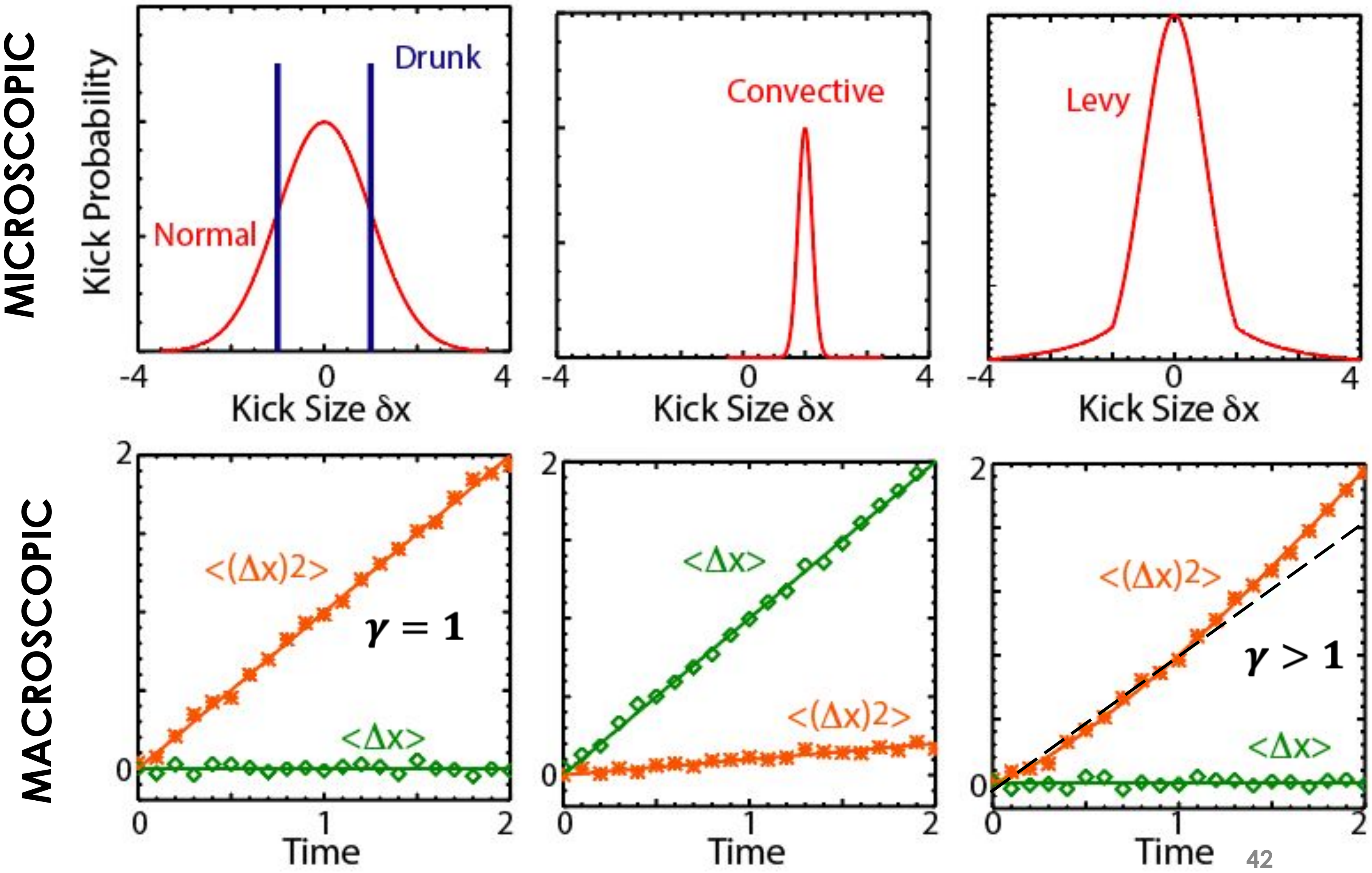
Random Walk: $D \approx (\delta x)^2/(2\delta t)$

Diffusion: $\langle(\Delta x)^2\rangle = 2Dt^{\gamma}$ **with** $\gamma = 1$

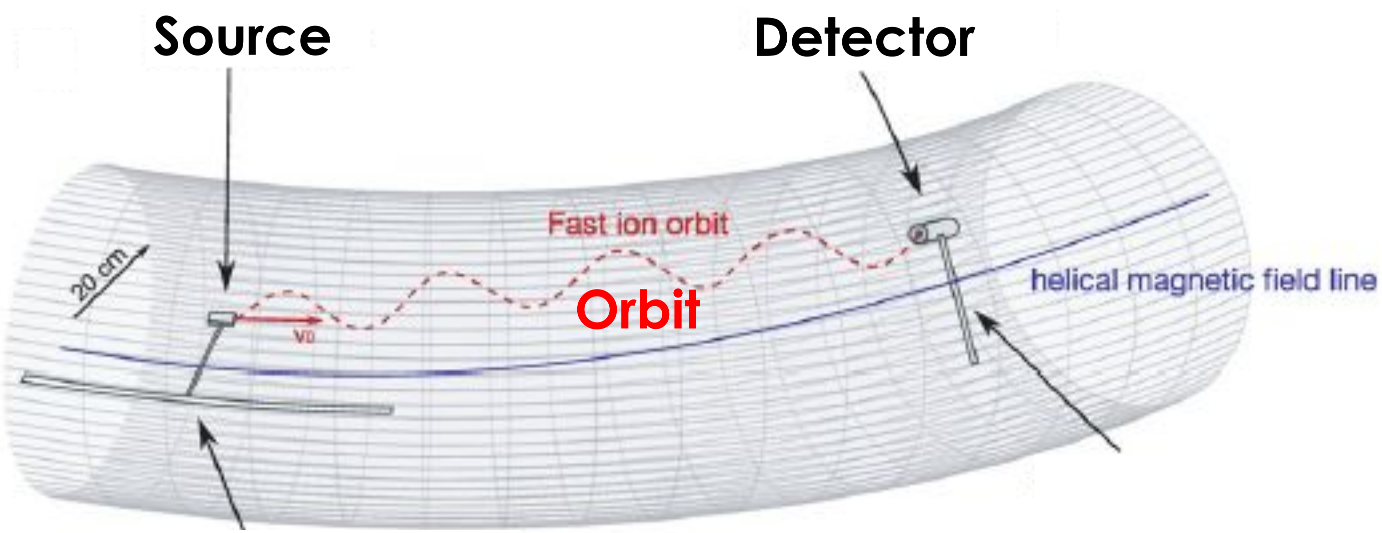
The microscopic kicks determine the macroscopic behavior

$$\langle(\Delta x)^2\rangle = 2Dt^{\gamma}$$

$\gamma = 1$ **"Diffusive"**
$\gamma > 1$ **"Super-Diffusive"**
$\gamma < 1$ **"Sub-Diffusive"**

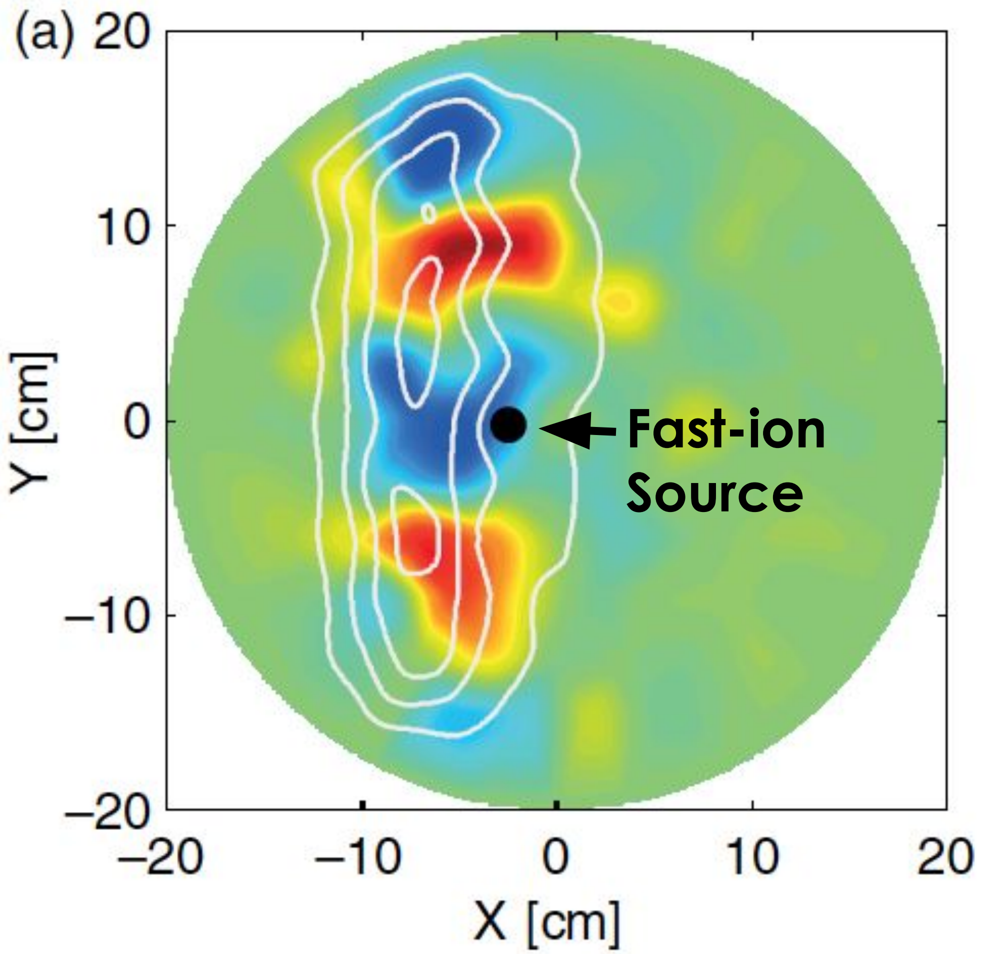


Sub-diffusive and super-diffusive EP transport was measured on the small toroidal device TORPEX



- Source launches collimated beam of fast ions
- Detector measures their spreading
- Electrostatic "blobs" cause beam spreading

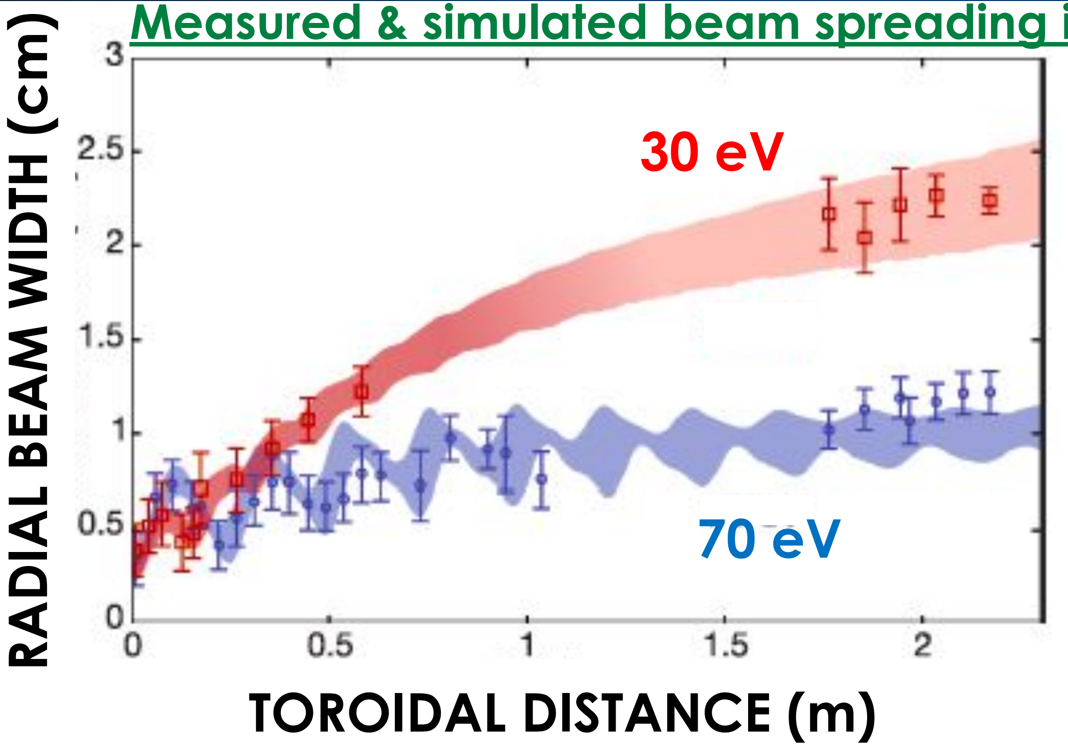
Measured fluctuations in TORPEX

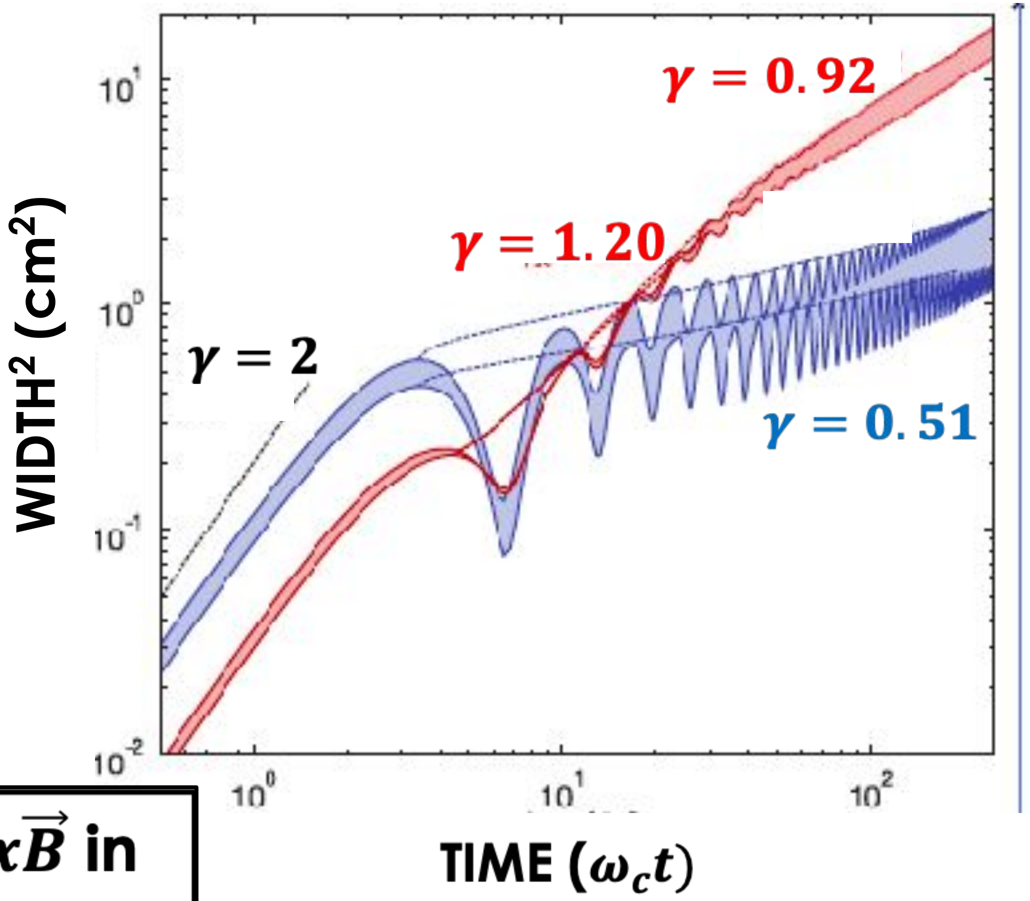


Bovet, Nucl. Fusion 52 (2012) 094017

Sub-diffusive and super-diffusive EP transport was measured on the small toroidal device TORPEX

Measured & simulated beam spreading in TORPEX





- Lower energy: Long transport steps by $\vec{E}x\vec{B}$ in blob → Levy flight → super-diffusive
- Higher energy: Weak interaction with blobs → sub-diffusive

Bovet, Phys. Rev. E 91 (2015) 041101

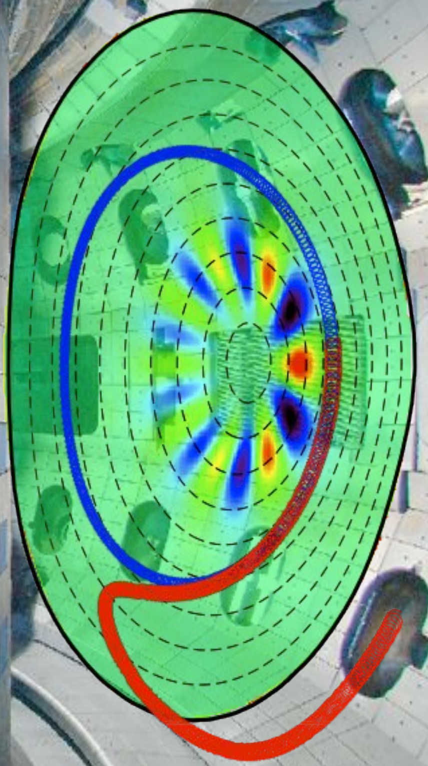
Outline

1. Orbits are described by constants of motion

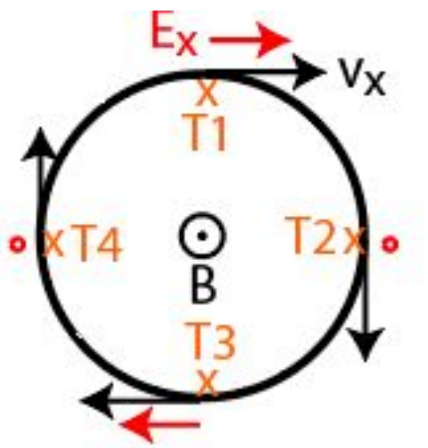
2. Perturbations break constants-of-motion

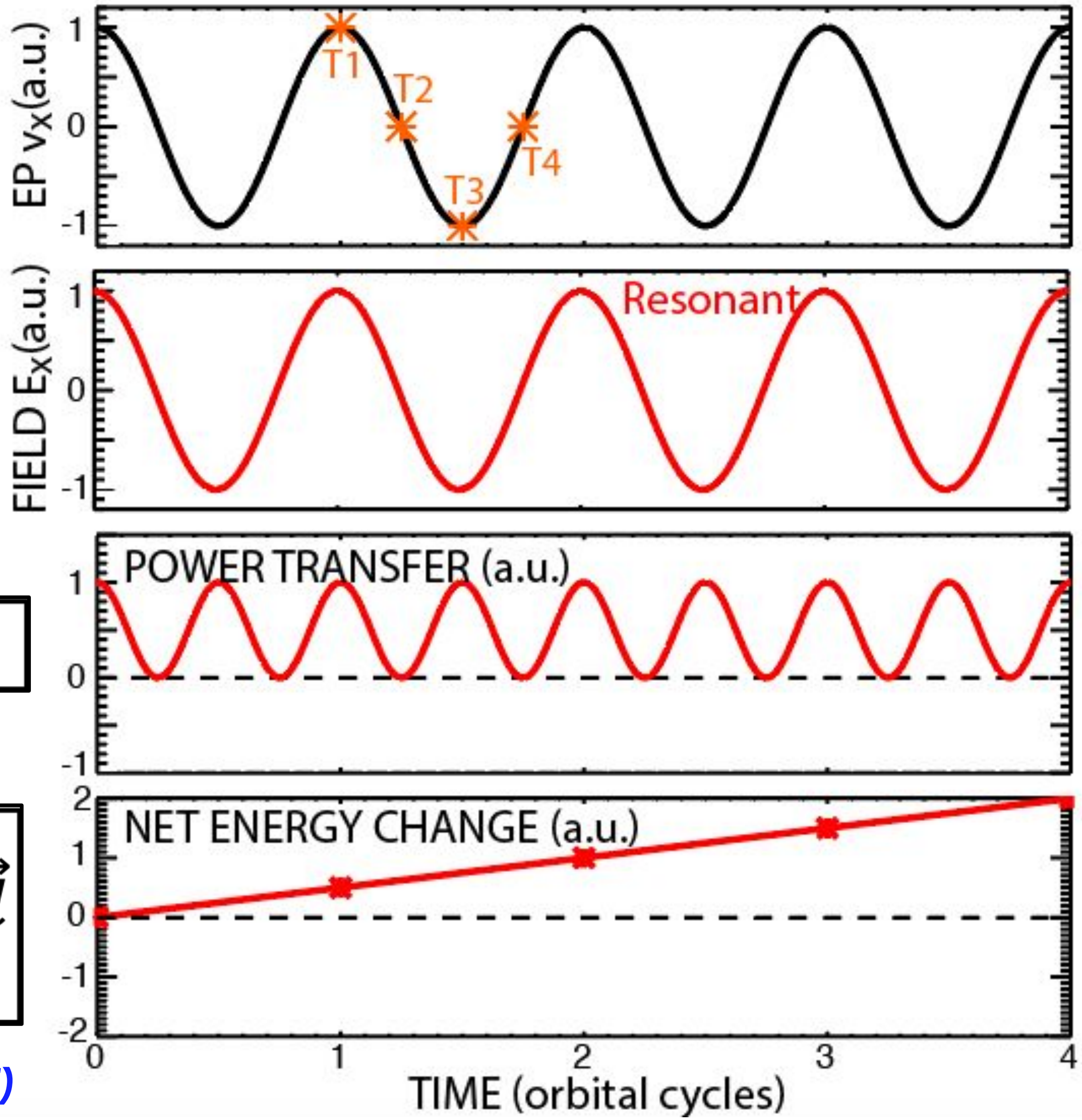
3. The wave-particle phase determines the type of transport

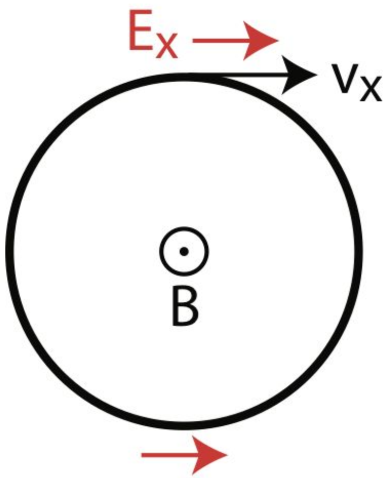
- Resonance occurs when the particle and wave return to the same initial phase
- The energy exchanged depends upon $\oint \vec{E} \cdot \mathrm{d}\vec{l}$ integrated over the cycle
- Non-resonant particles are weakly affected
- EPs that are slightly off resonance are nonlinearly trapped by the wave



Resonance occurs when the particle and the wave return to the same initial phase after a cycle





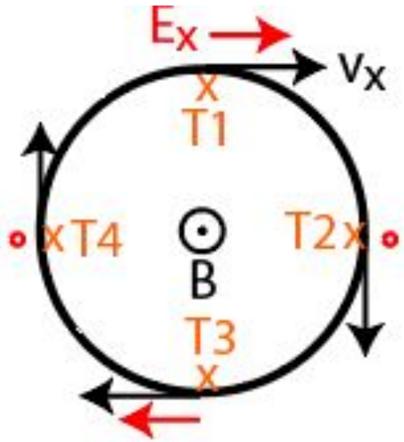


$$P = dW/dt = q\vec{E} \cdot \vec{v}$$

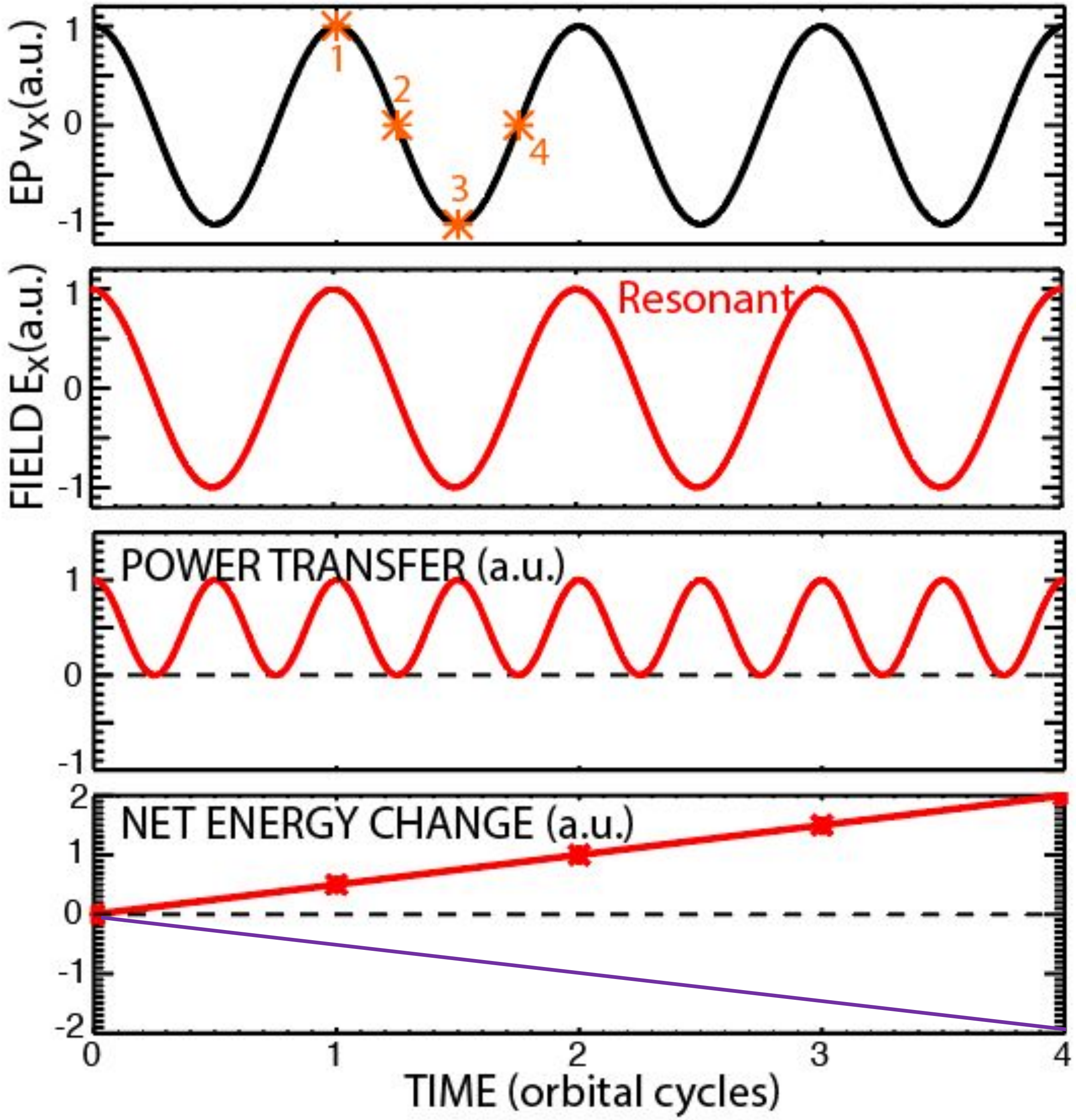
$$\Delta W = q \oint \vec{E} \cdot d\vec{l}$$

Pace, Physics Today (October 2015)

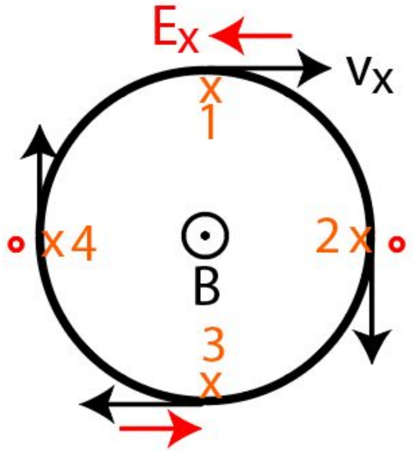
Resonance occurs when the wave frequency matches an orbital frequency



- $\mu = W_\perp / B$ is <u>not</u> conserved
- The opposite initial phase <u>loses</u> energy—still a resonance that breaks μ

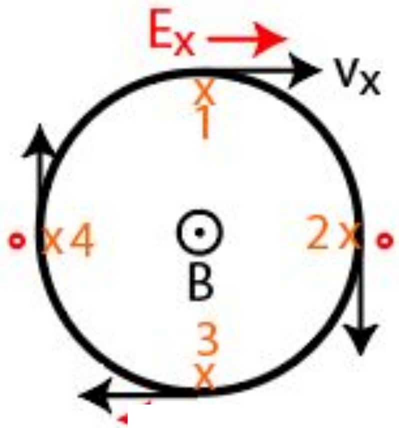


Opposite Phase

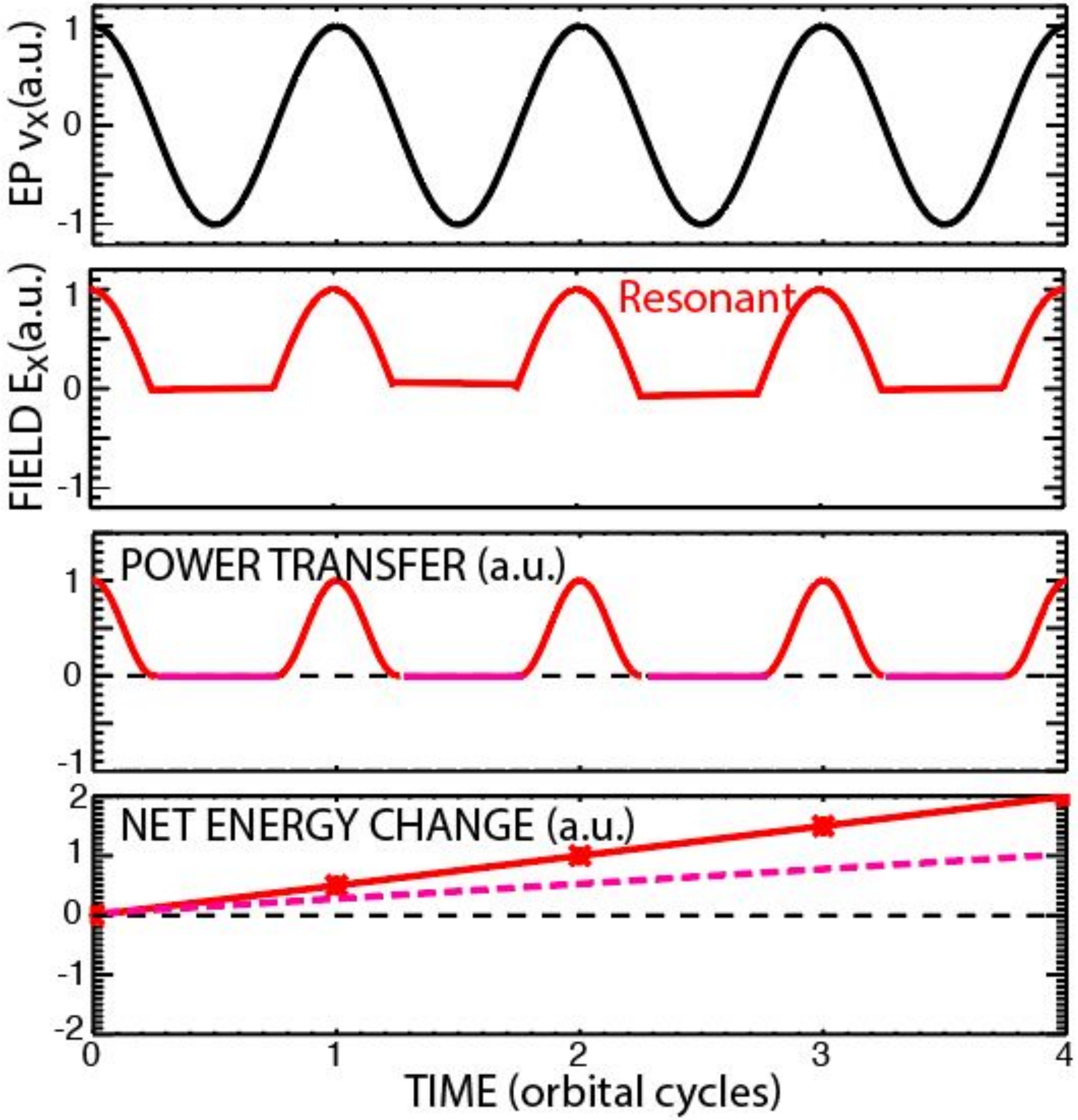


Pace, Physics Today (October 2015)

The particle does not need to gain energy on the entire orbit

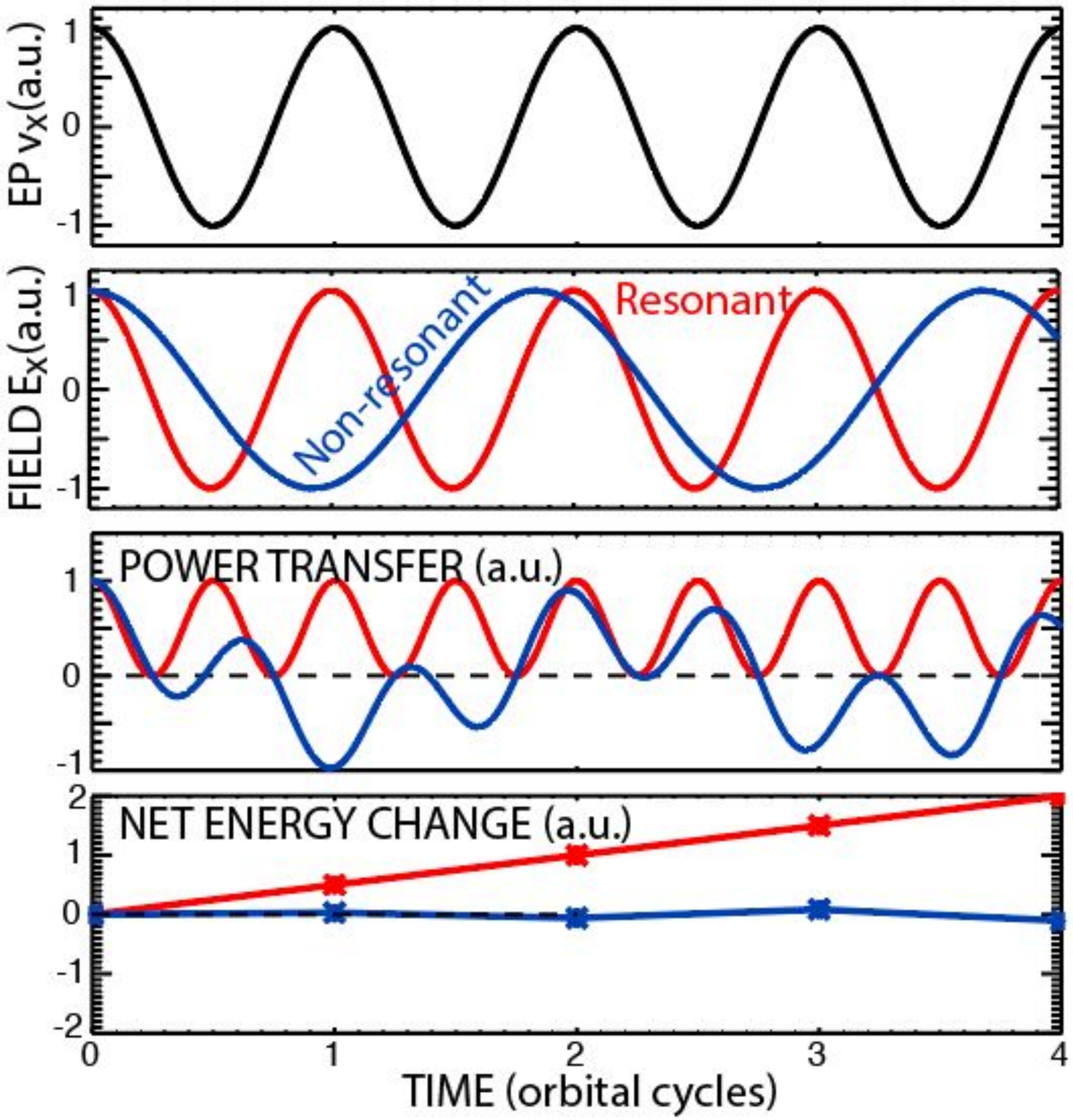


- **If the field was only in the upper half, the EP is still resonant**
- **Energy transfer requires** $\oint \vec{E} \cdot d\vec{l} \neq 0$



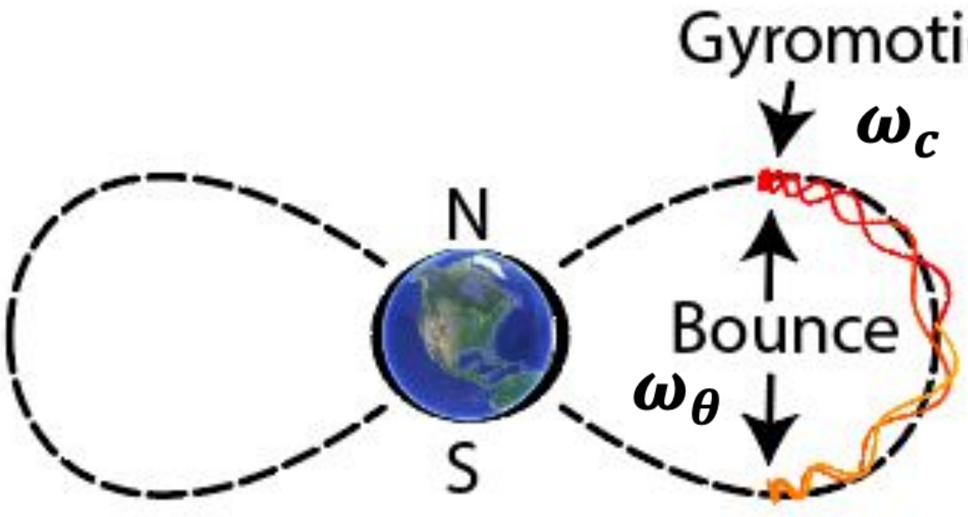
A non-resonant particle does not gain net energy

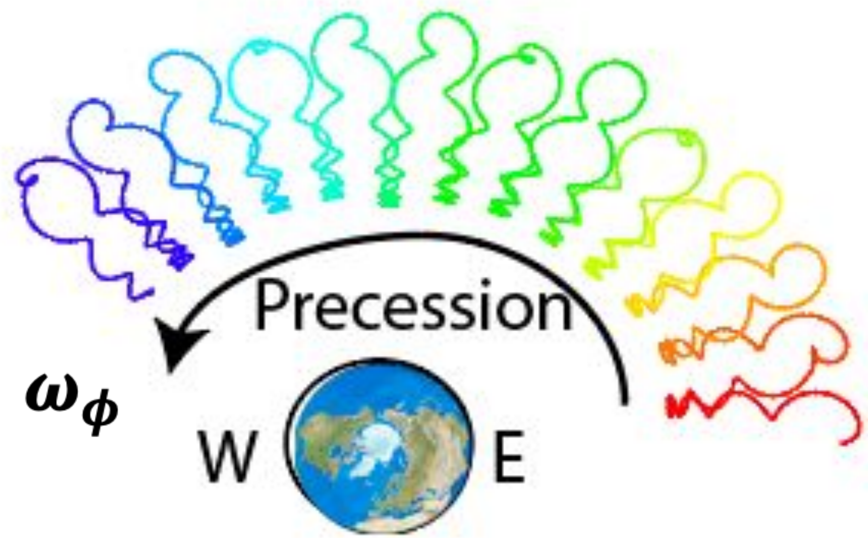
- ***μ*** **<u>is</u> conserved**



Pace, Physics Today (October 2015)

Resonance can be with any aspect of the orbital motion



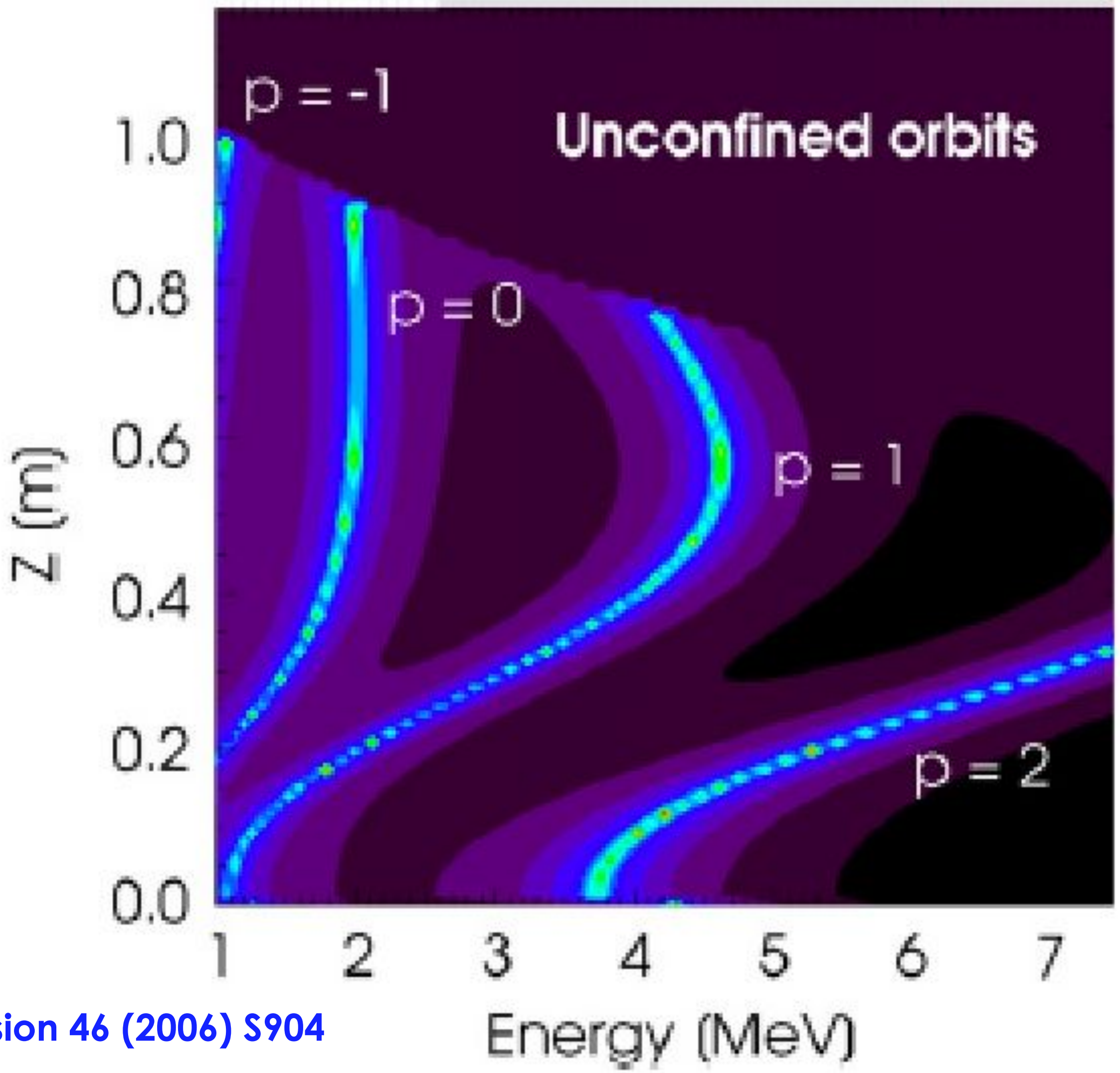


$$\omega = \ell\omega_c + p\omega_\theta + n\omega_\phi$$

l, p, n are integers

A perturbation can resonate with many different orbits

Resonances of an Alfven wave with RF-tail ions in the JET tokamak



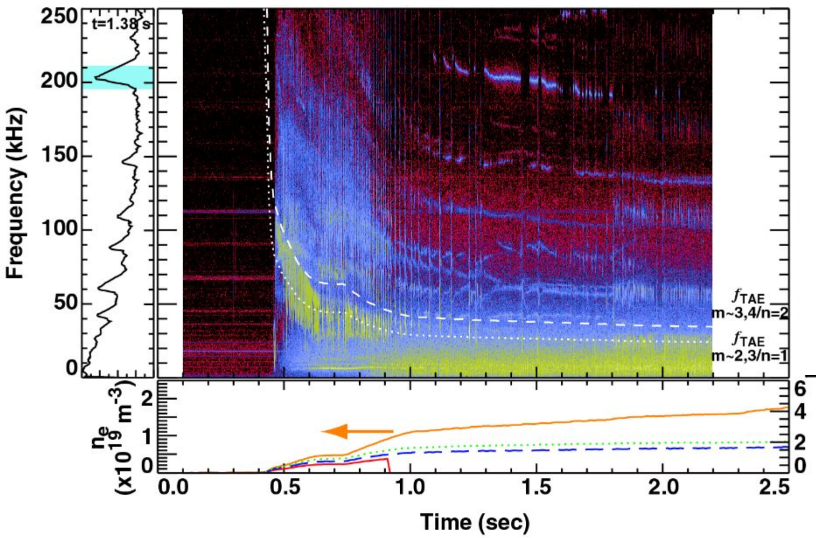
$$\omega = \ell\omega_c + p\omega_\theta + n\omega_\phi$$

(with $\ell\omega_c = 0$ and $n\omega_\phi = 0$)

Pinches, Nucl. Fusion 46 (2006) S904

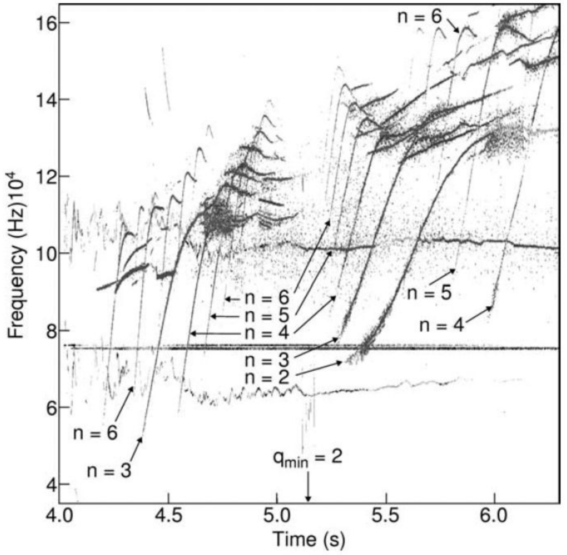
Tremendous variety of resonances are observed

Beam injection, CHS stellarator[1]



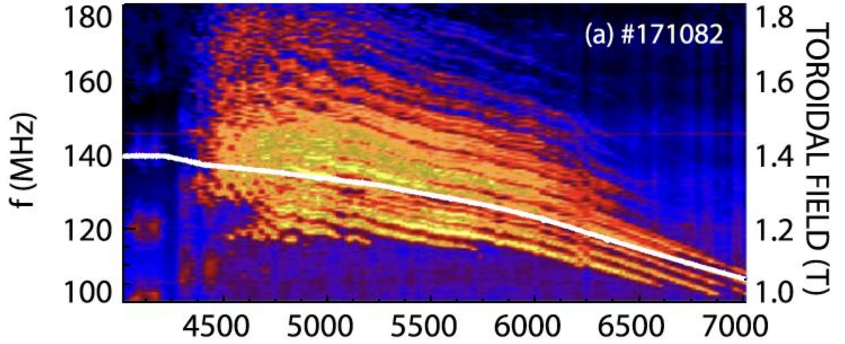
HAE & TAEs

RF tail ions, JET tokamak[2]



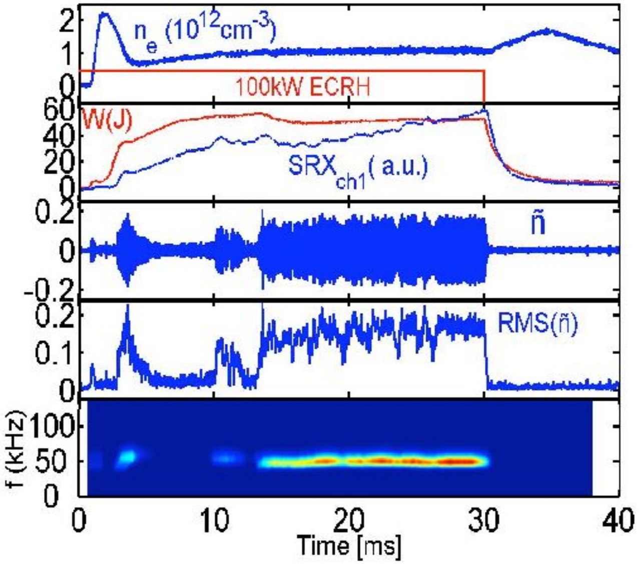
TAEs & RSAEs

Runaways, DIII-D tokamak[5]



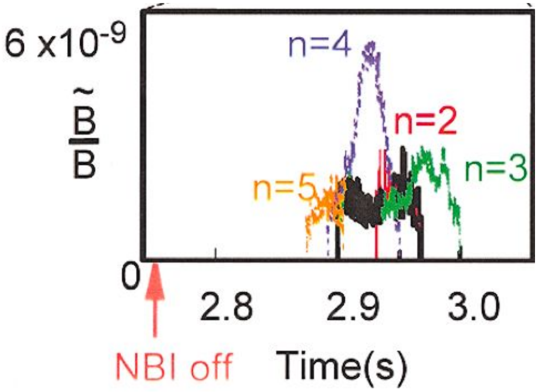
Whistler

Electron tail, HSX stellarator[3]



GAE

Alphas, TFTR tokamak[4]

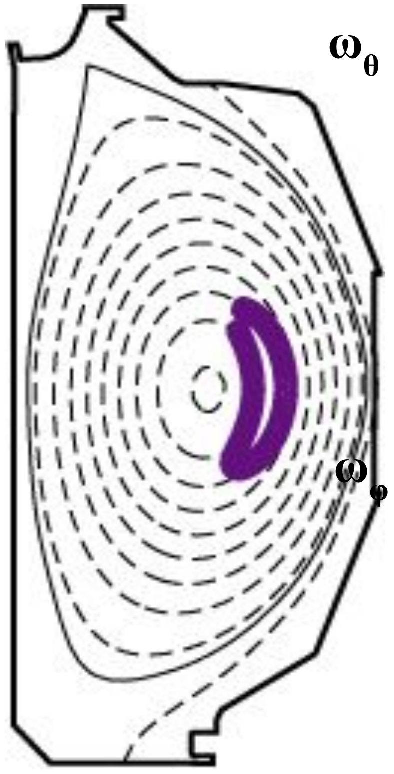


TAEs & RSAEs

[1]Yamamoto, PRL 91 (2003) 245001; [2]Sharapov, Phys. Lett. A 289 (2001) 127; [3]Brower; [4]Nazikian, PRL 78 (1997) 2976; Heidbrink, PPCF 61 (2018) 014007

For Alfvén waves, the strength of the resonance depends upon $\oint \vec{v_d} \cdot \vec{E}$

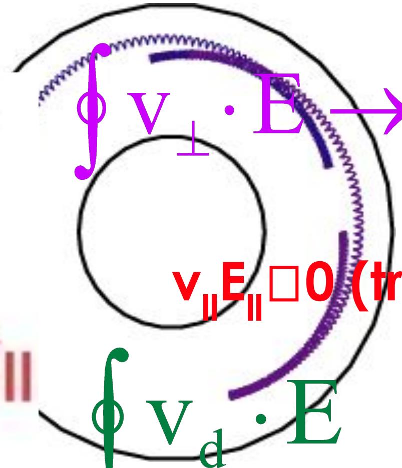
$$\omega = \ell\omega_c + p\omega_\theta + n\omega_\phi$$



ω_θ

ω_φ

Time to comple
poloidal
□ ω_θ



$\oint v_\perp \cdot E \rightarrow 0 \; (\Omega_c >> \omega)$

Time to complete toroidal orbit
□ ω_φ

$v_{\|} E_{\|}$□0 (transverse polarization)

$\oint v_d \cdot E$ (main energy exchange)

Write v_d as a Fourier expansion in terms of poloidal angle θ: $\sum_{l=\pm1,\pm2,...} A_l e^{il\theta}$

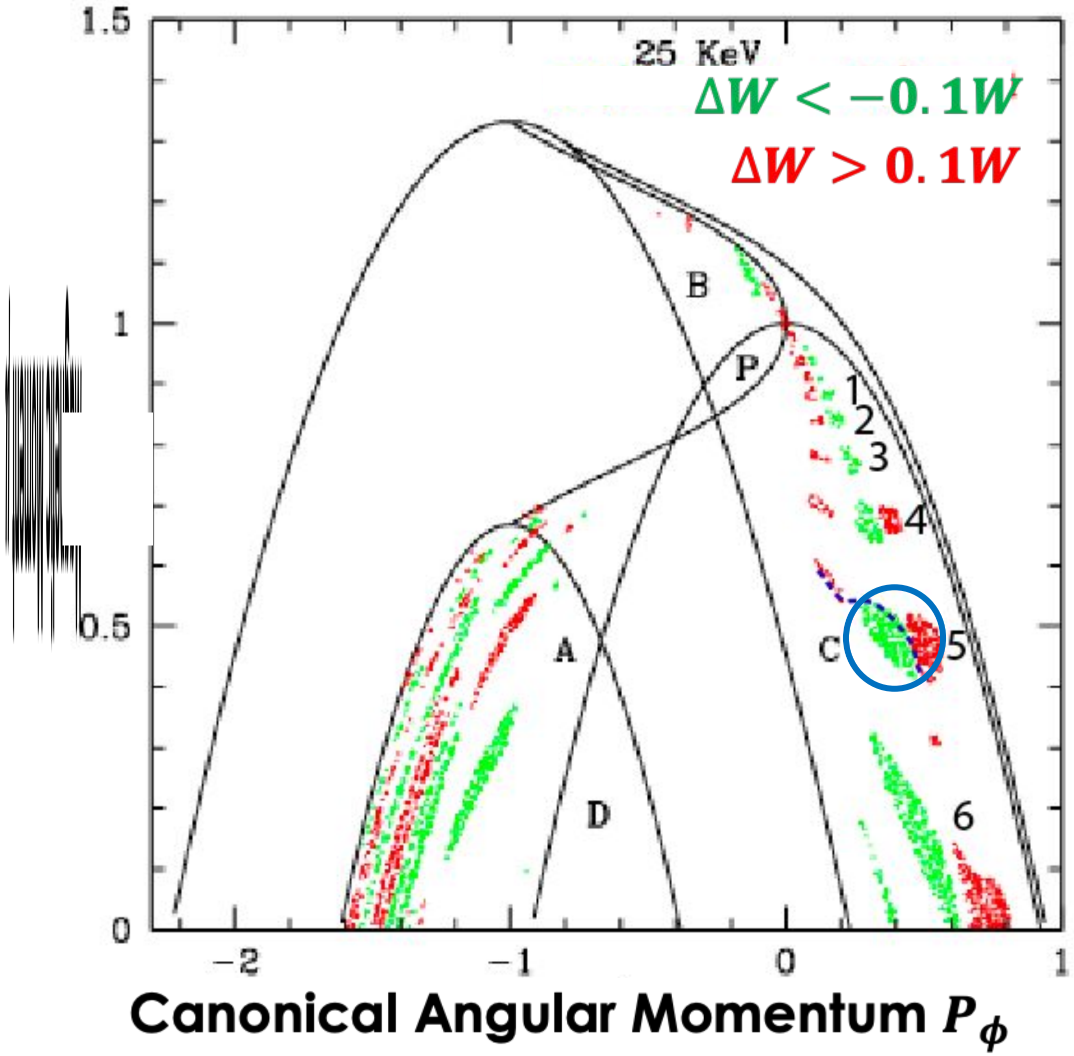
Energy exchange resonance condition: $\omega = (m+l)\omega_\theta + n\omega_\phi$

Wave mode #s

Drift harmonic

EPs that are slightly off resonance are nonlinearly trapped by the wave

<u>Calculated energy exchange of beam ions with an Alfven wave in DIII-D</u>



- Colors indicate energy exchange
- Many harmonics are important

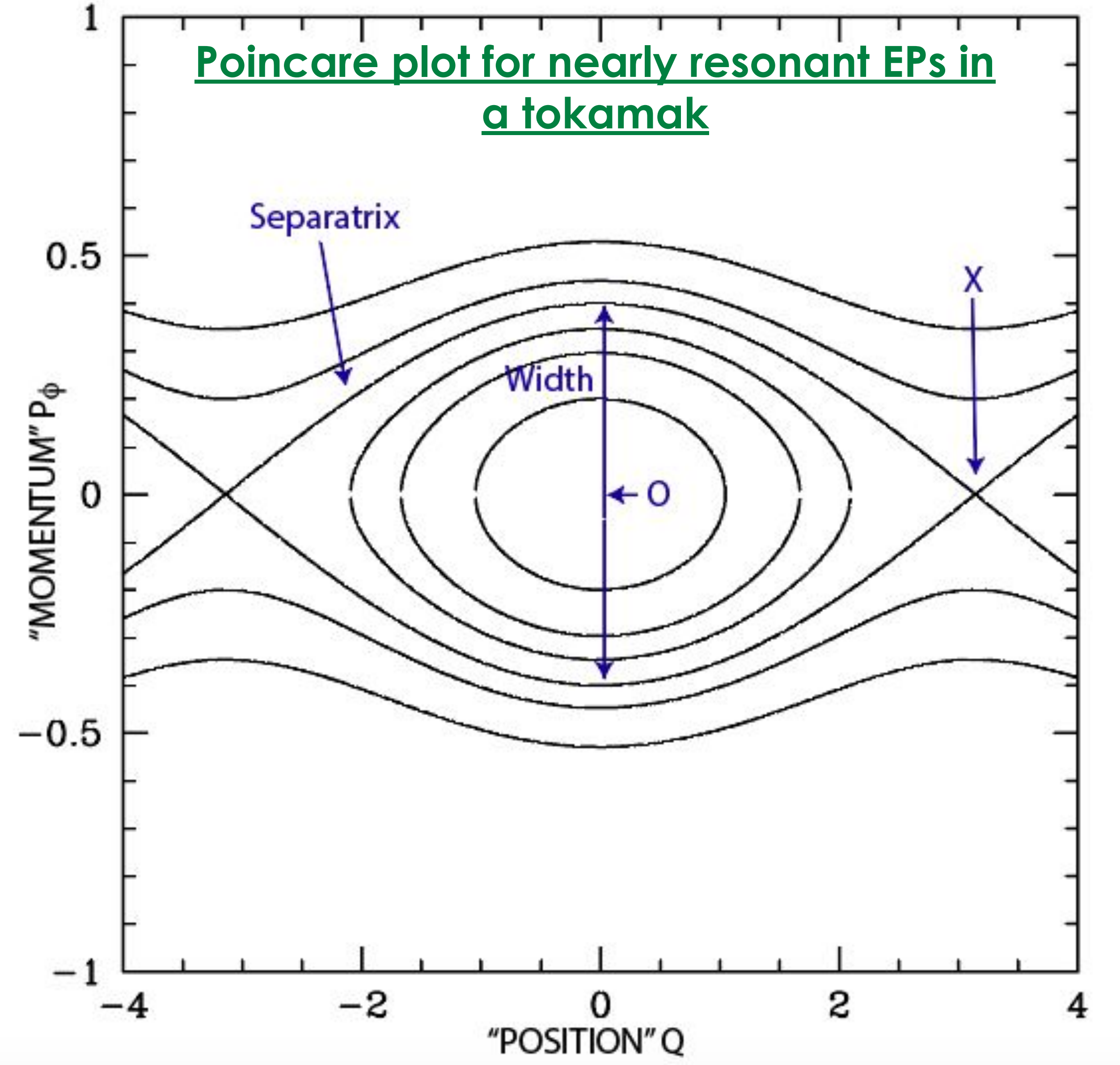
$= 0$ (for ℓ), $= 2$ (for n); p is circled

$$\omega = \ell\omega_c + p\omega_\theta + n\omega_\phi$$

$$\omega \approx \ell\omega_c + p\omega_\theta + n\omega_\phi$$

White, Plasma Phys. Cont. Fusion 52 (2010) 045012

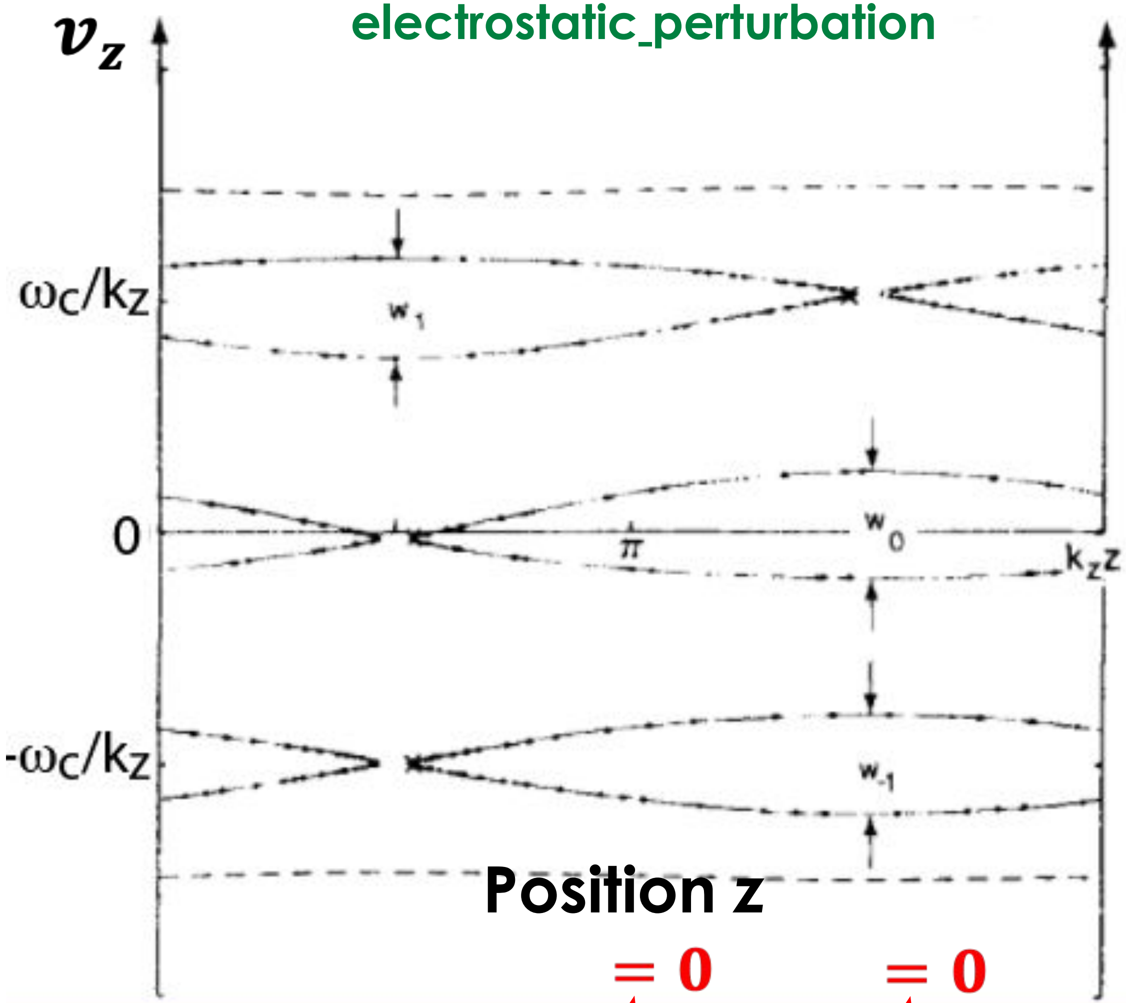
Nearly resonant EPs are trapped by a finite amplitude wave



- Particles at "X" and "O" points satisfy resonance exactly
- Particles inside "separatrix" are trapped & travel with wave
- Width of trapping region $\propto \sqrt{Amplitude}$

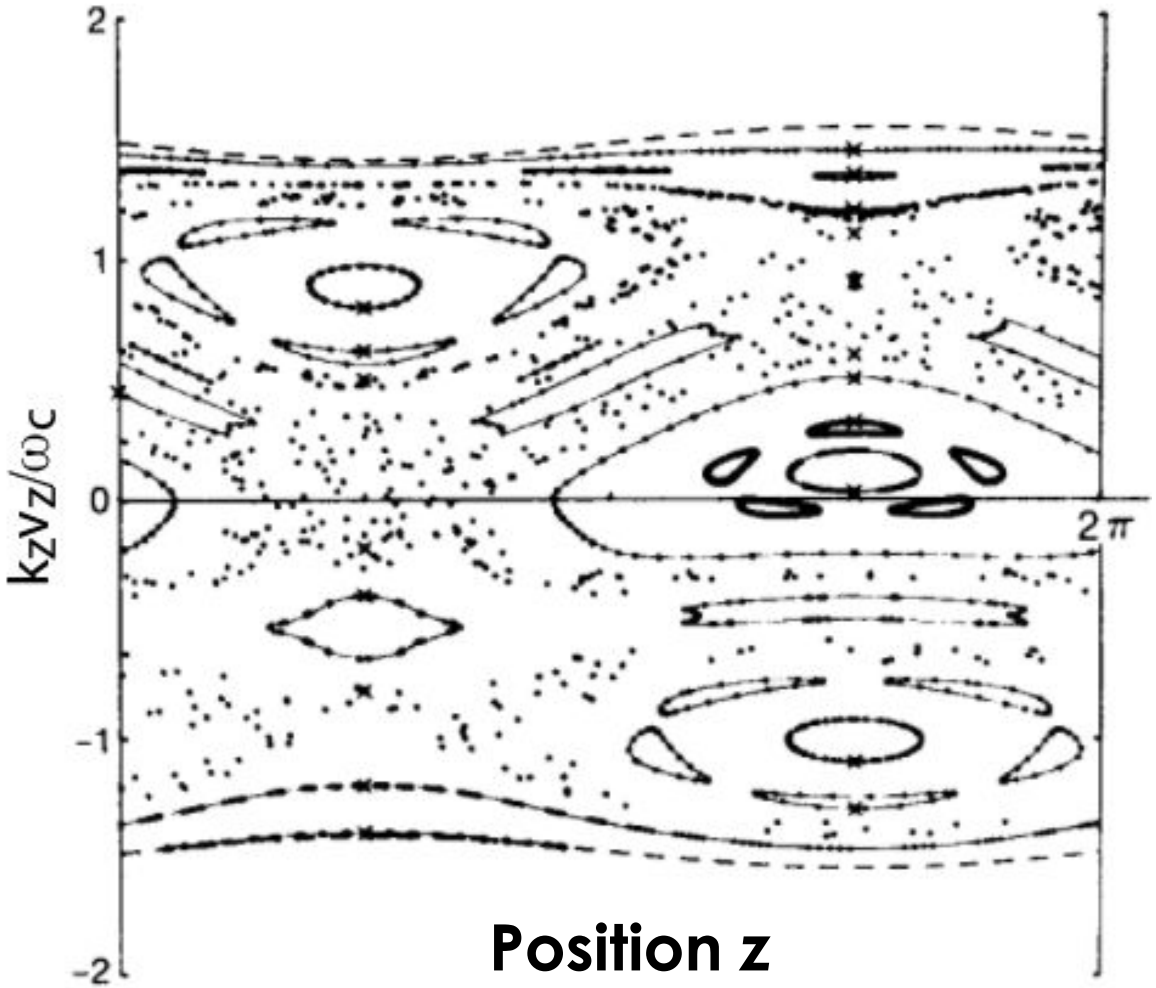
Orbits become stochastic when resonances overlap

Calculated island widths of orbits in small electrostatic perturbation



$$\omega = \ell\omega_c + m\omega_\theta + n\omega_\phi$$

Calculated orbits in large perturbation

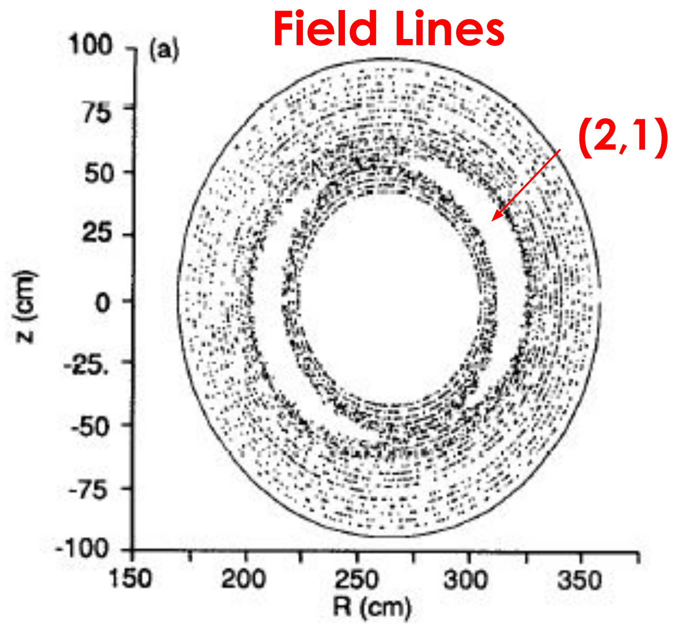


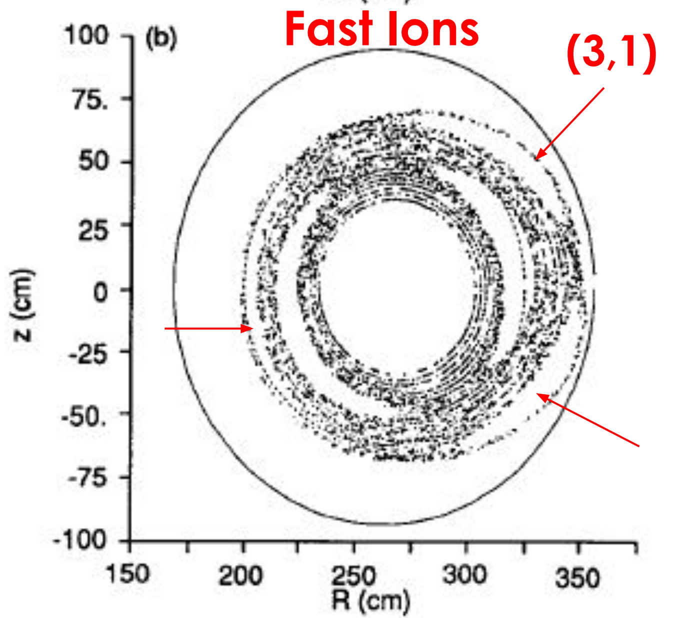
Smith, Phys. Fluids 21 (1978) 2230

EP orbits create additional island chains

- A tearing mode creates an (m,n)=(2,1) perturbation to the axisymmetric tokamak field
- A co-passing fast ion experiences the curvature drift that causes an (m,n)=(1,0) shift of the orbit from the flux surface
- Beating between the two perturbations causes a (3,1) island chain

If the (2,1) field perturbation is large & the ion is energetic, the islands overlap □ stochastic transport □ ion is rapidly lost

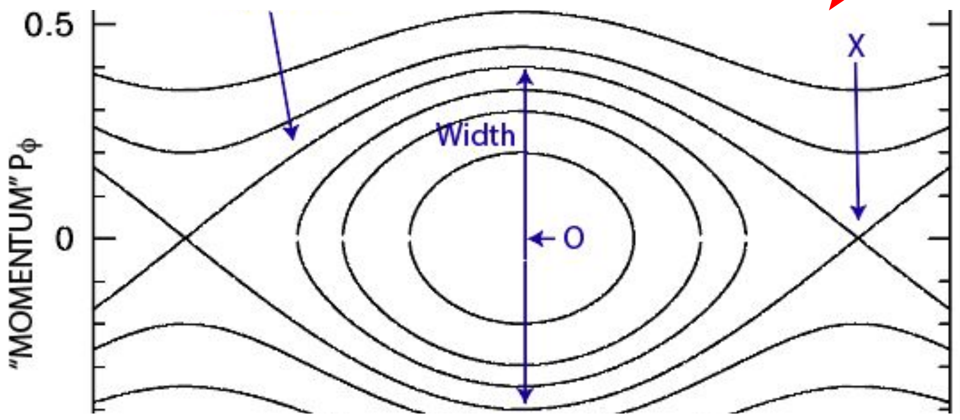


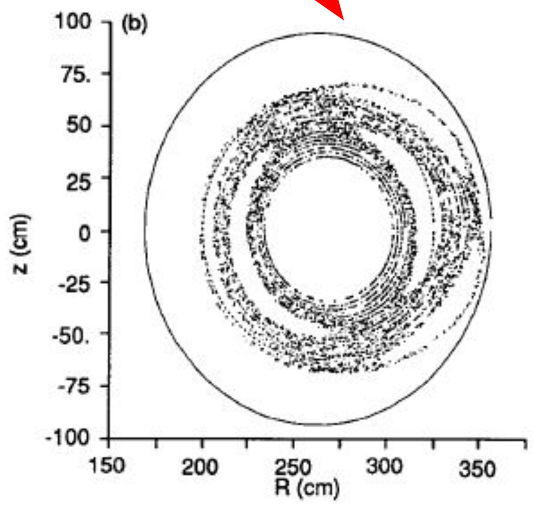


Mynick, Phys. Fluids B 5 (1993) 2460

Which statement about wave-particle resonance is <u>false</u>?

1. Resonance occurs when the wave frequency matches integer multiples of orbital frequencies $\omega = \ell\omega_c + p\omega_\theta + n\omega_\phi$
2. Some resonances exchange more energy than others $\oint \vec{E} \cdot d\vec{l} \neq 0$
3. The resonance condition does <u>not</u> need to be matched exactly if the wave has a finite amplitude.
4. EP orbits can be stochastic even when the field lines are not.
5. None of these (they are all true).

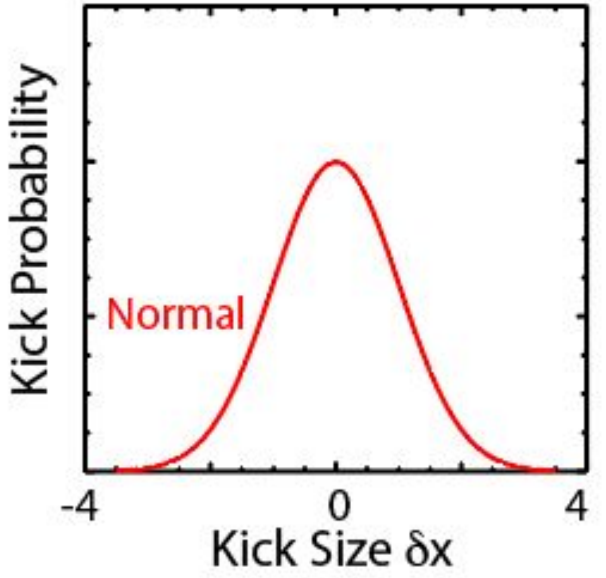


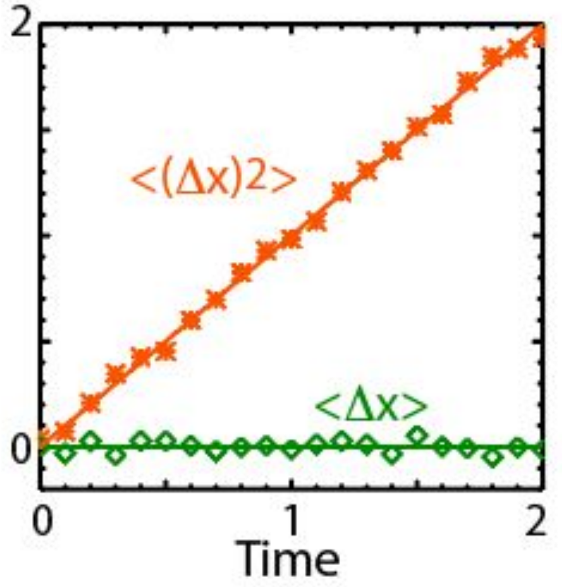


Outline

1. Orbits are described by constants of motion
2. Field perturbations break constants-of-motion & cause transport
3. The wave-particle phase determines the type of transport
4. **For non-resonant modes, orbit averaging dramatically reduces transport**

- **Fast-ion example**
- **Runaway electron example**

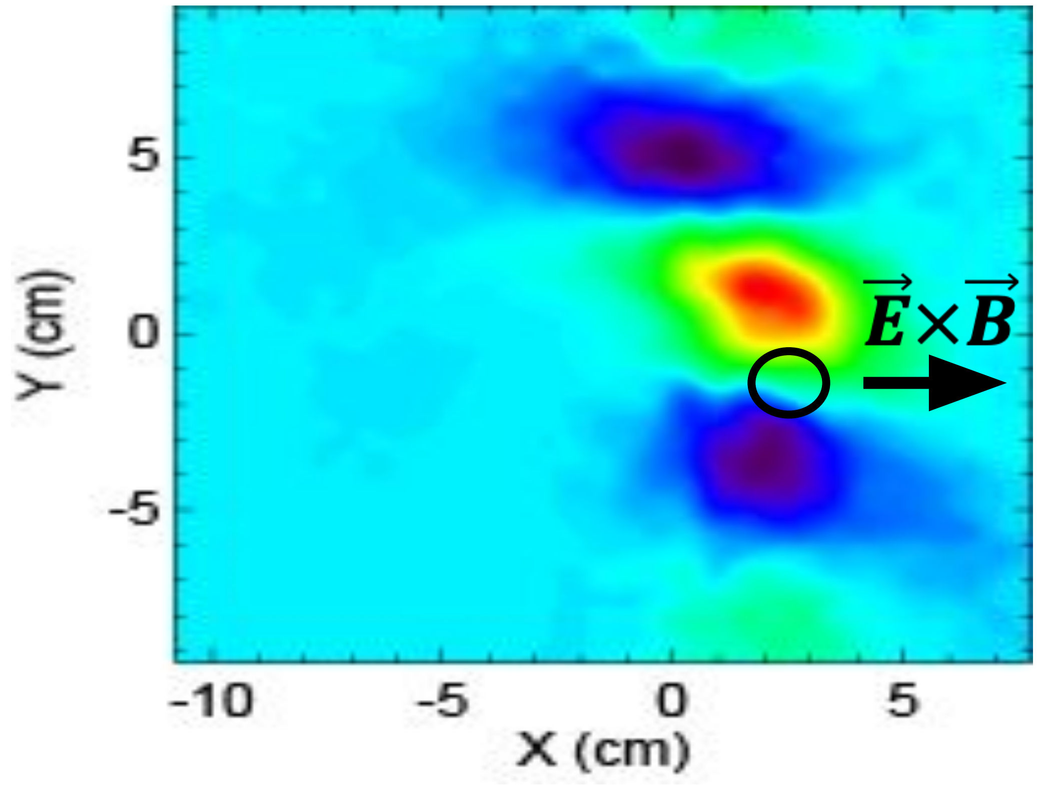




Large orbits spatially filter electrostatic turbulence

Drift wave created by an obstacle in the linear LAPD

Fluctuation Amplitude φ



$$\Delta W = q\oint \vec{E}\cdot d\vec{l}$$

- Potential fluctuations in plane perpendicular to B
- Small-orbit ion stays in phase with wave □ large E x B kick

Zhou, Phys. Pl. 17 (2010) 092103; 19 (2012) 055904

Large orbits spatially filter electrostatic turbulence

Drift wave created by an obstacle in the linear LAPD

Fluctuation Amplitude Φ

$$\Delta W = q \oint \vec{E} \cdot d\vec{l}$$

- Potential fluctuations in plane perpendicular to B
- Small-orbit ion stays in phase with wave → large E x B kick
- Large orbits "see" different phases of $\vec{E}$

Zhou, Phys. Pl. 17 (2010) 092103; 19 (2012) 055904

Large orbits spatially filter electrostatic turbulence

Drift wave created by an obstacle in the linear LAPD

Fluctuation Amplitude Φ

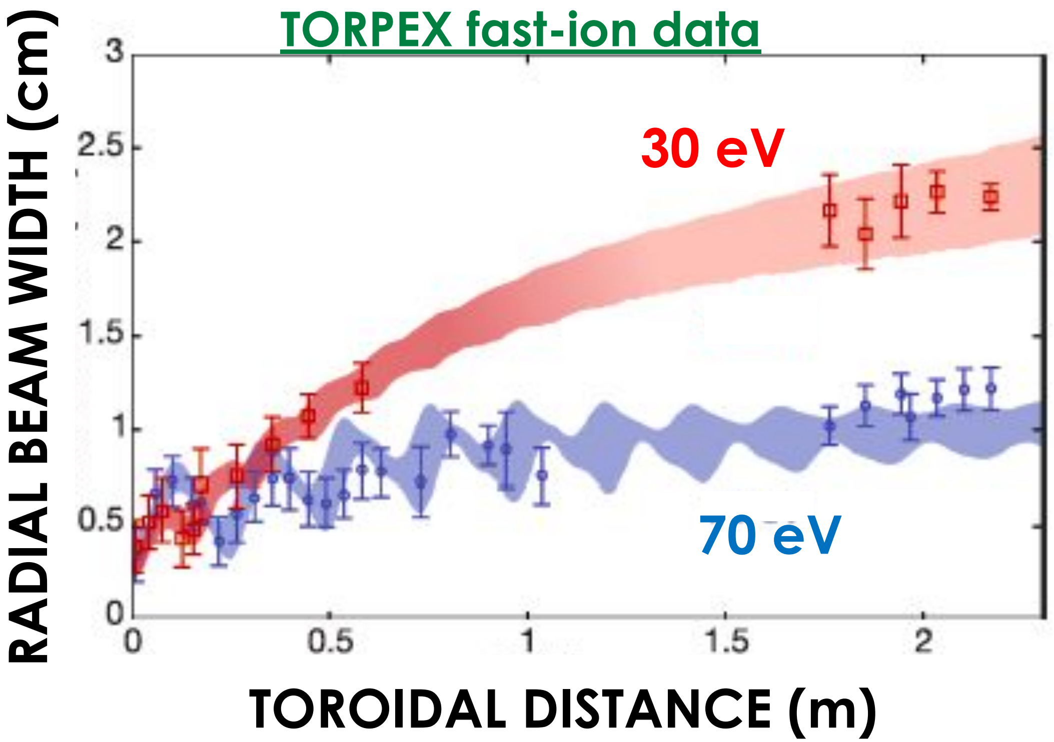
- "Phase averaging" reduces transport
- $\oint \vec{E} \cdot d\vec{l} \propto J_o(k_\perp \varrho)$

Beam Spreading (cm^2)

Fast-ion Energy (eV)

Zhou, Phys. Pl. 17 (2010) 092103; 19 (2012) 055904

The higher energy beam spreads less because of phase averaging



Bovet, Phys. Rev. E 91 (2015) 041101

Runaway electron diffusion due to δB stochasticity

Field lines diffuse due to electromagnetic turbulence

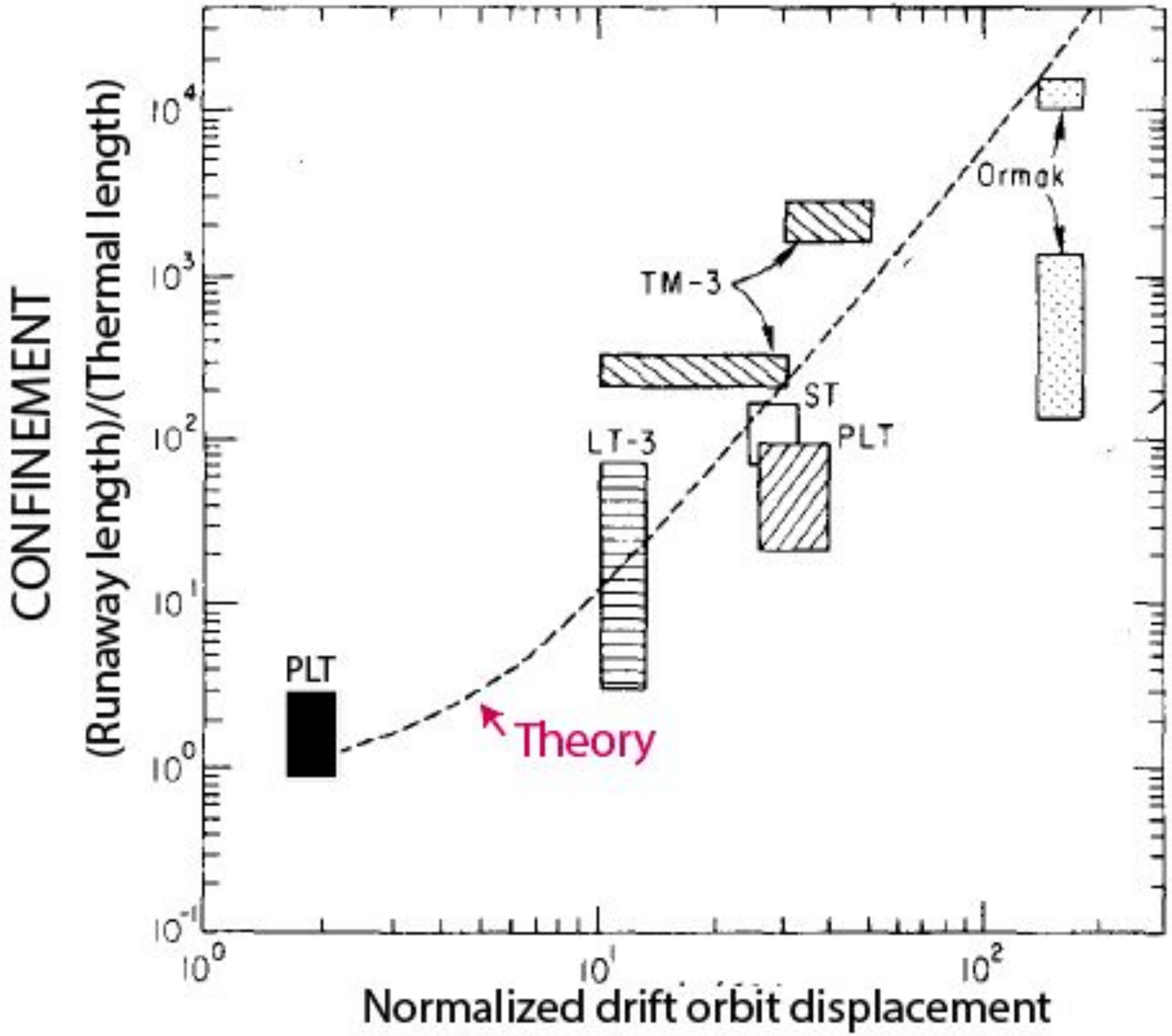
$\boldsymbol{v}_{\|}$ $\quad$ $\boldsymbol{D} \propto \boldsymbol{v}_{\|}$

Rechester, PRL 40 (1978) 38

□ Confinement degrades with energy

Runaway electron diffusion due to δB turbulence: Phase averaging reduces transport at large energy

Runaway confinement data from many tokamaks



- Large curvature drift makes orbit size > than scale length of turbulent fluctuations
- Similar result for beam ions in the Reversed Field Pinch MST

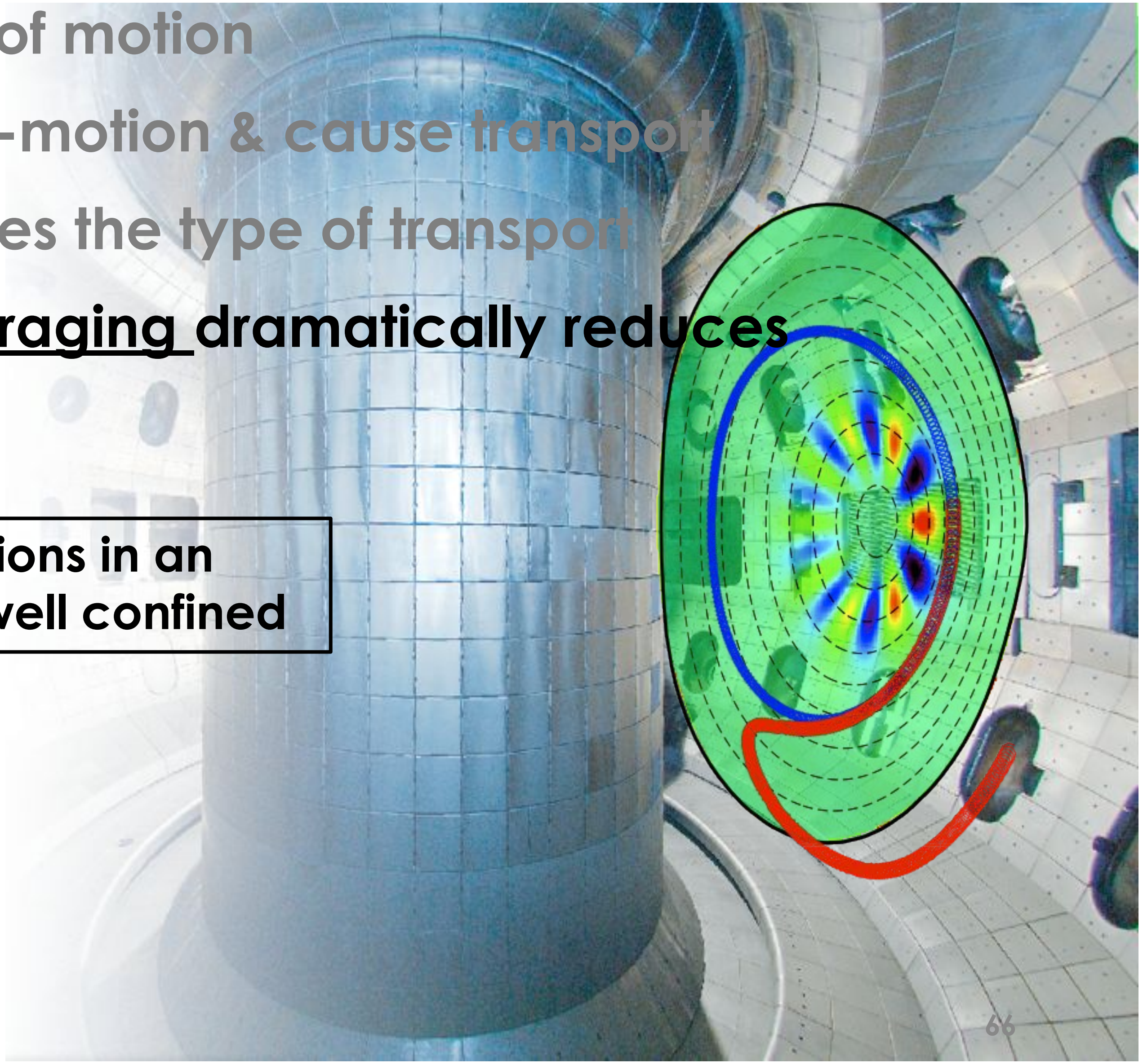
Fiksel PRL 95 (2005) 125001.

Mynick, Phys. Fluids 24 (1981) 695.

Outline

1. Orbits are described by constants of motion
2. Perturbations change constants-of-motion & cause transport
3. The wave-particle phase determines the type of transport
4. **For non-resonant modes, orbit averaging dramatically reduces transport**

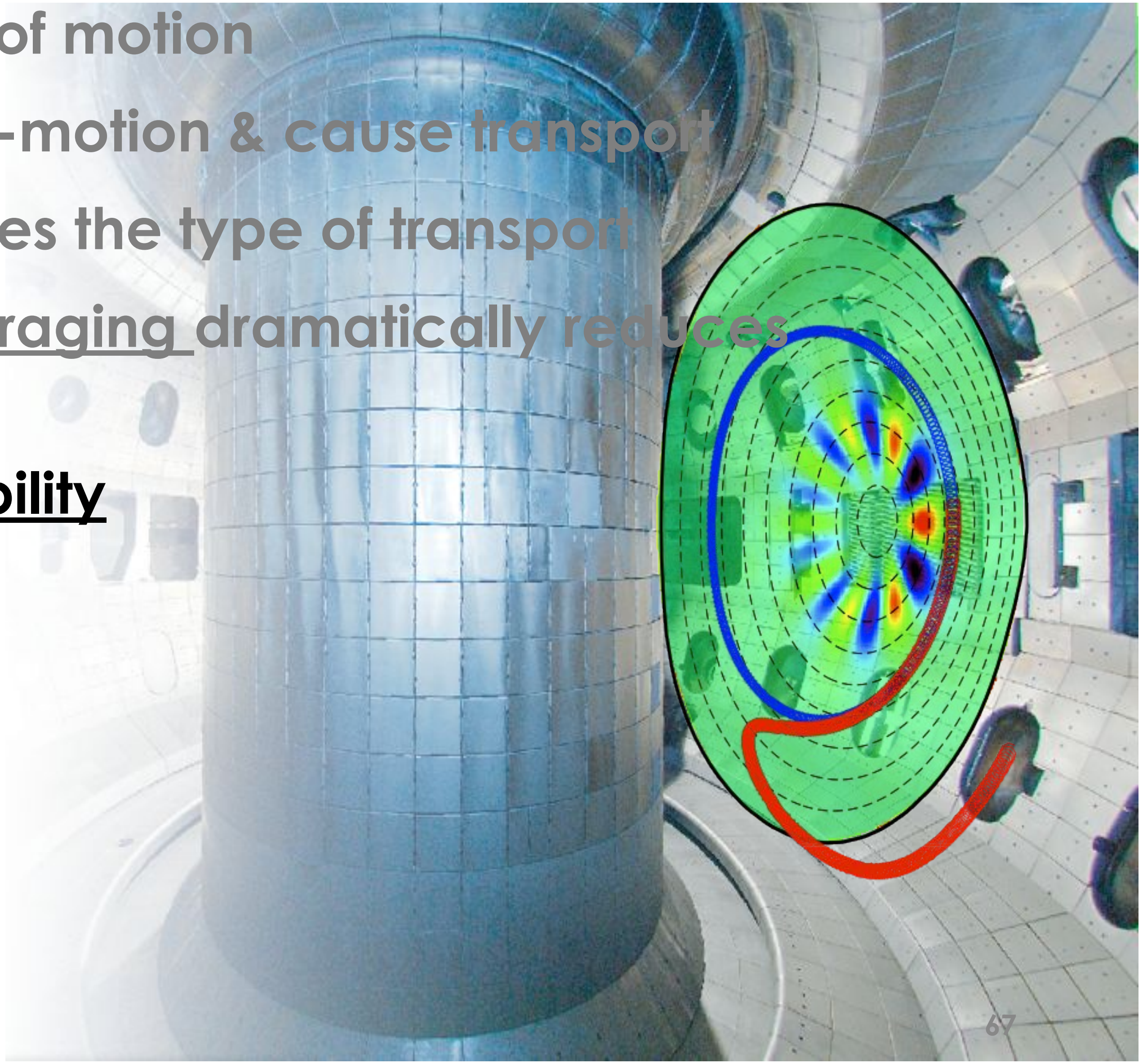
Without resonant instabilities, fast ions in an axisymmetric tokamak are very well confined



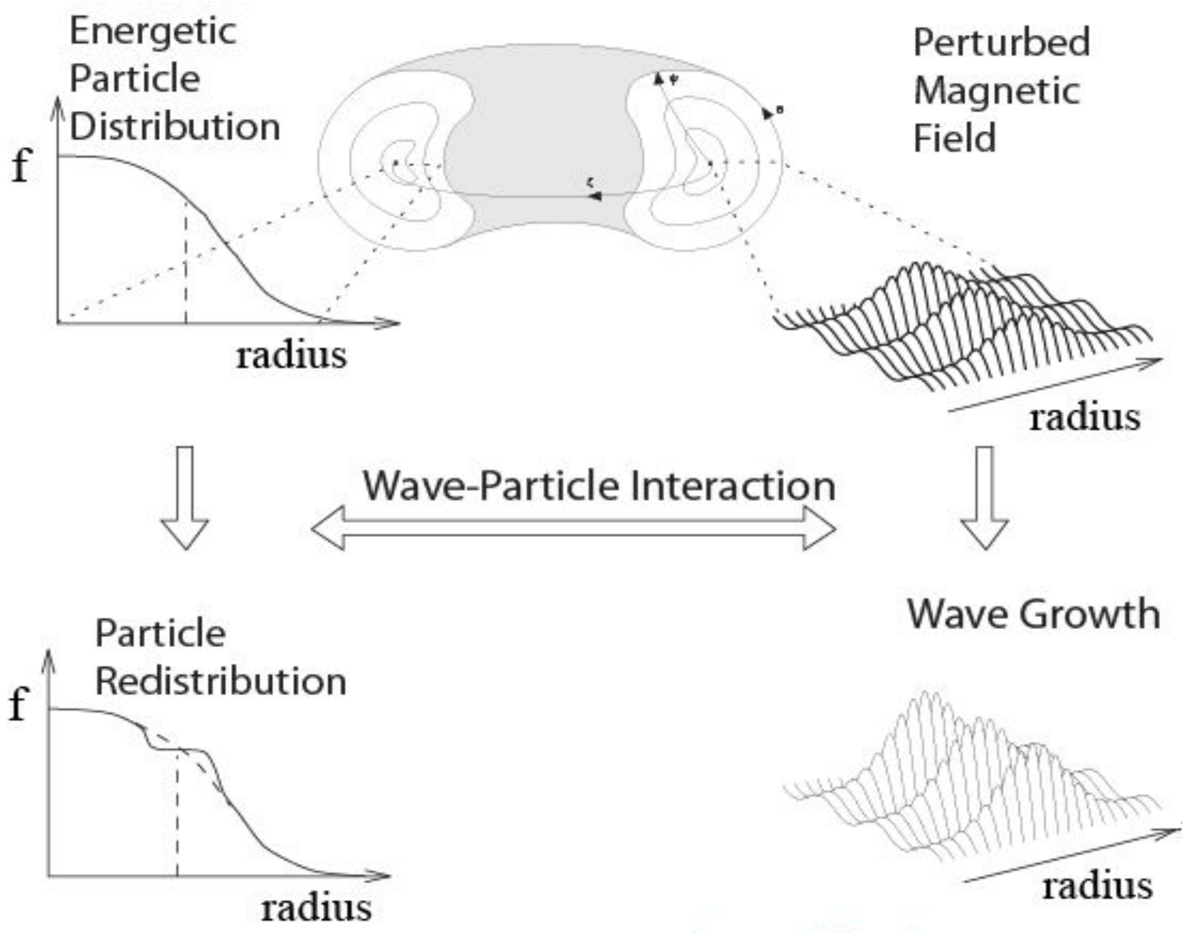
Outline

1. Orbits are described by constants of motion
2. Perturbations change constants-of-motion & cause transport
3. The wave-particle phase determines the type of transport
4. For non-resonant modes, orbit averaging dramatically reduces transport
5. **Resonant particles can drive instability**

- **Gradients drive instability**
- **Alfvén eigenmodes are unavoidable**



Instability involves a complex interplay between the distribution function and the waves



Pinches, PhD Thesis

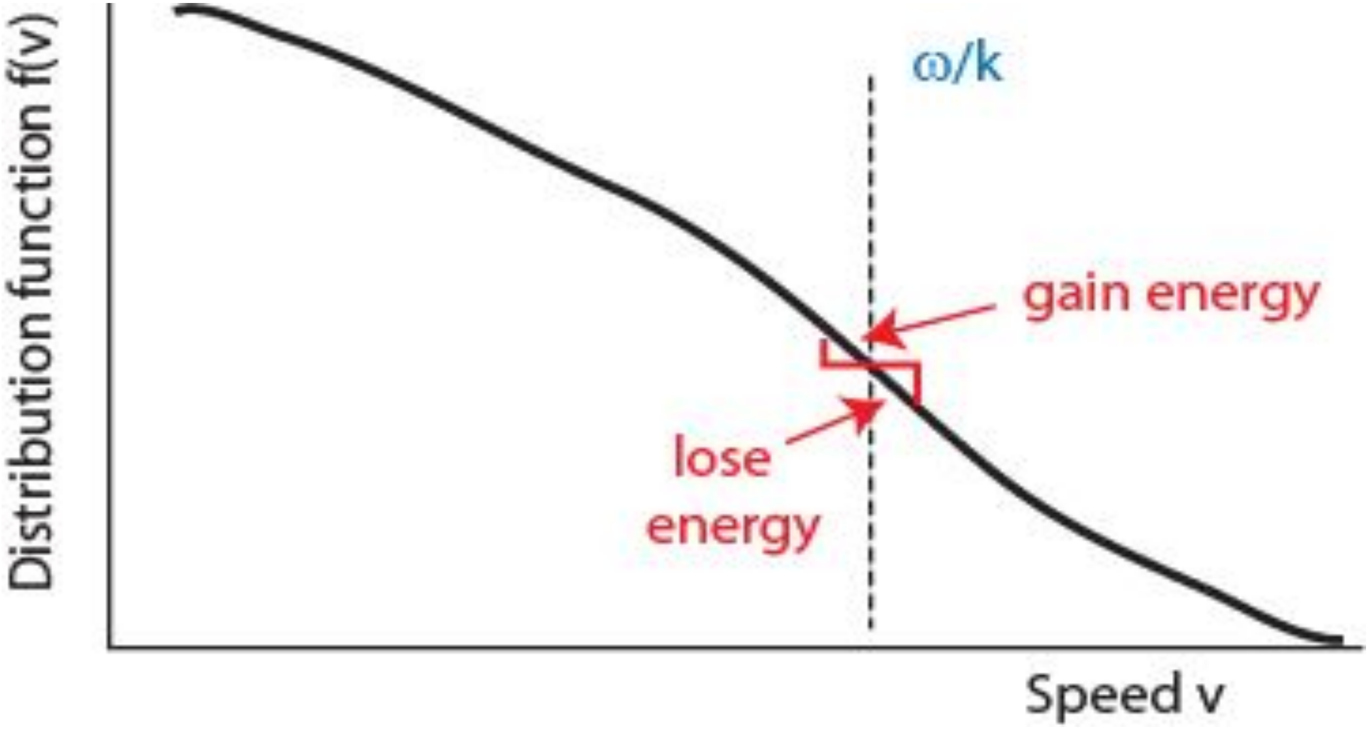
The slope of the distribution function at resonances determines whether waves damp or grow

- Nearly resonant particles are trapped in finite amplitude wave
- After trapping, slower ones have gaine energy; faster ones have lost
- If $\partial f / \partial v < 0$ the wave damps

- All gradients contribute in general

$$\gamma \sim \omega \partial f / \partial W, n \partial f / \partial P_{\phi}, \partial f / \partial \mu$$

Landau damping

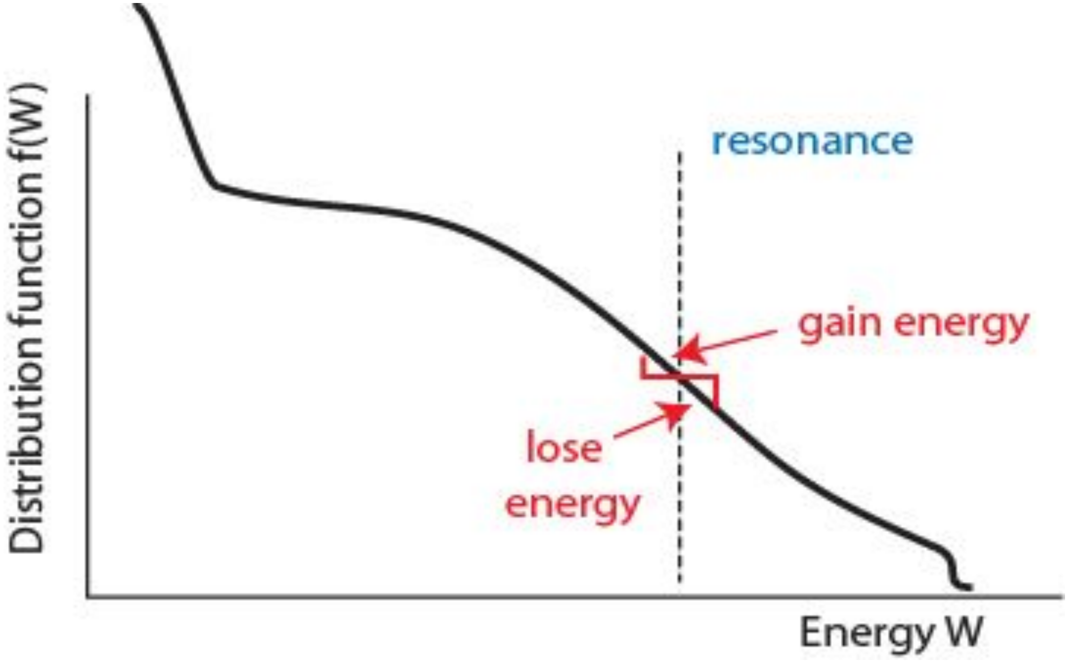


The spatial gradient of the distribution usually drives instability

- All gradients contribute in general

$\gamma \sim \omega \partial f/\partial W, n\partial f/\partial P_\phi, \partial f/\partial \mu$

- Energy distribution usually decreases monotonically □ $\partial f/\partial W$ damps wave

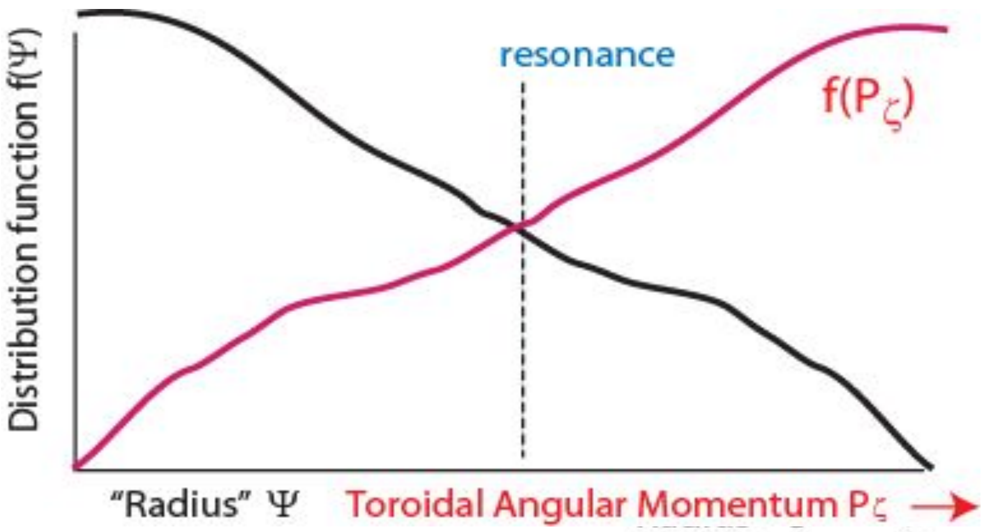


The spatial gradient of the distribution usually drives instability

- All gradients contribute in general

$\gamma \sim \omega \partial f/\partial W, n\partial f/\partial P_{\phi}, \partial f/\partial \mu$

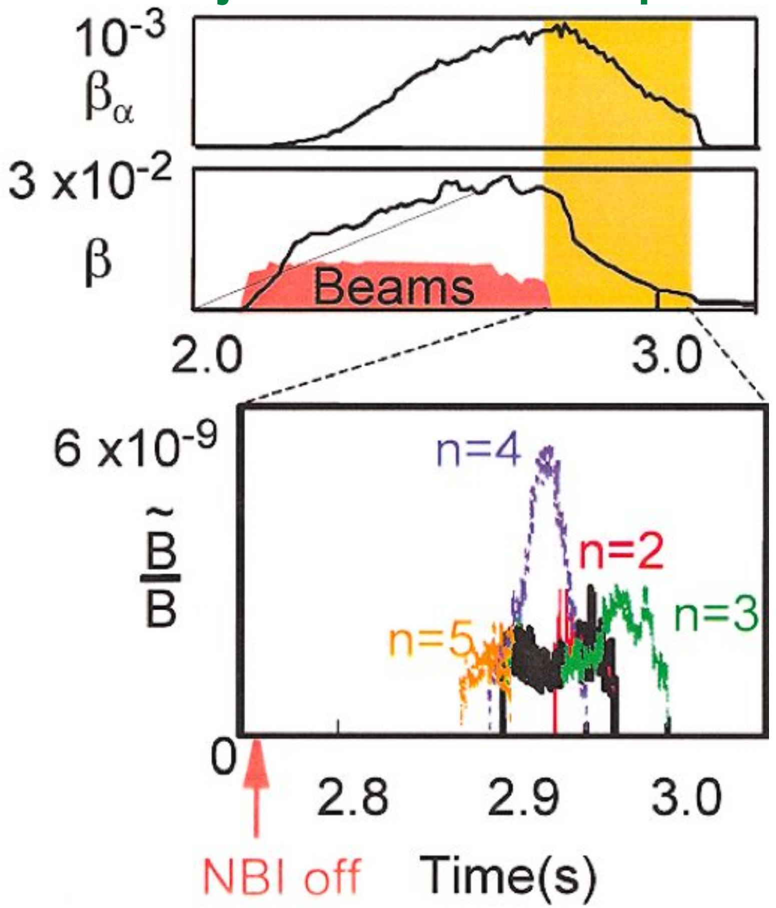
- Spatial distribution peaks on axis
- $P_{\varphi} = mRv_{\varphi} - Ze\Psi$ ($\Psi = RA_{\varphi}$ is the poloidal flux—a radial coordinate)



TAEs in TFTR: avoid energy damping by beam ions, use spatial gradient drive by alphas

- Strong $\partial f / \partial W$ beam-ion damping stabilized AEs during beam pulse
- Theory[1] suggested strategy to observe alpha-driven TAEs
- Beam damping decreased faster than alpha spatial gradient drive after beam pulse
- TAEs observed[2] when theoretically predicted

AEs observed after beam injection in TFTR D-T plasmas

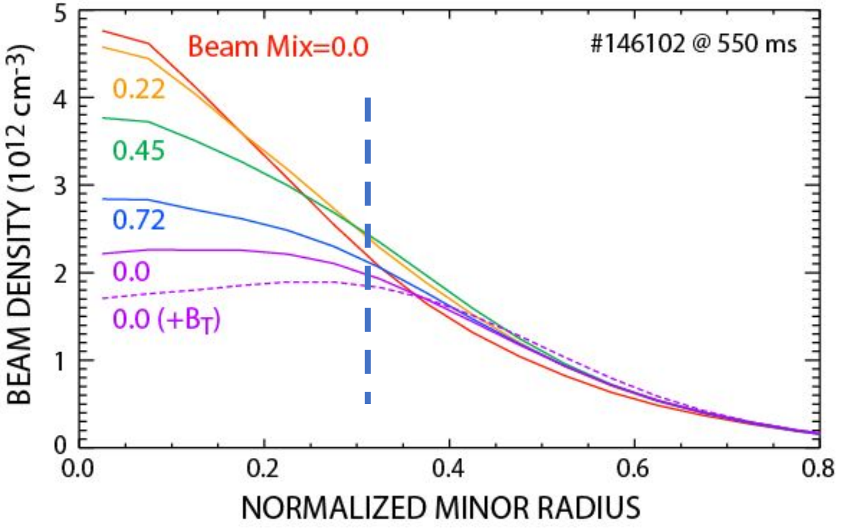


[1]Fu, Phys. Plasma 3 (1996) 4036;
Spong, Nucl. Fusion 35 (1995) 1687

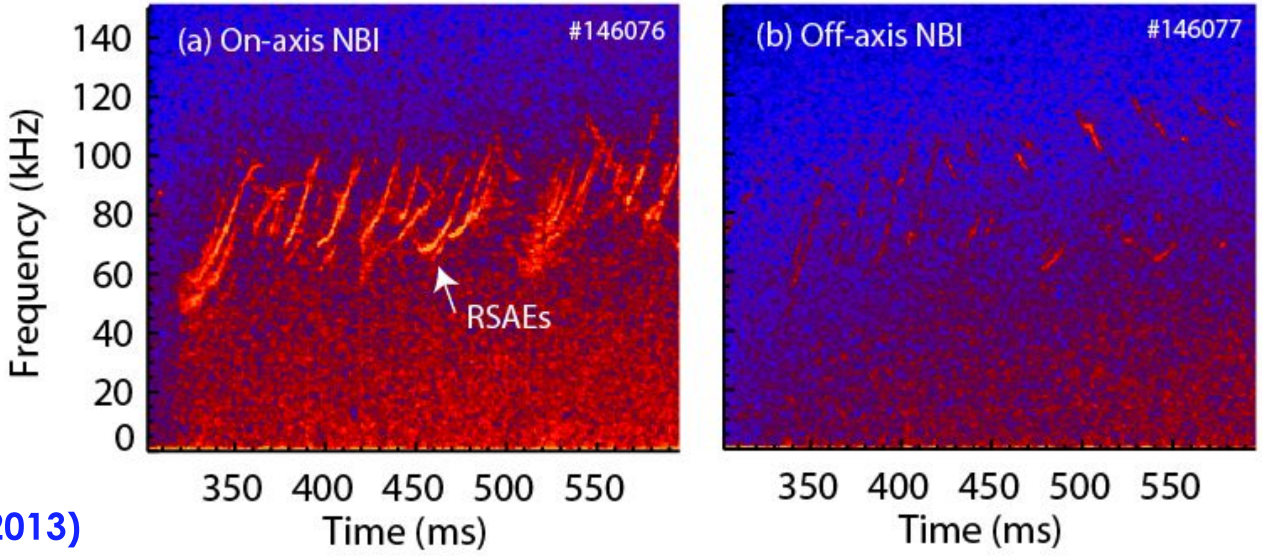
[2]Nazikian, PRL 78 (1997) 2976

The expected dependence on fast-ion gradient is observed experimentally

DIII-D Experiment



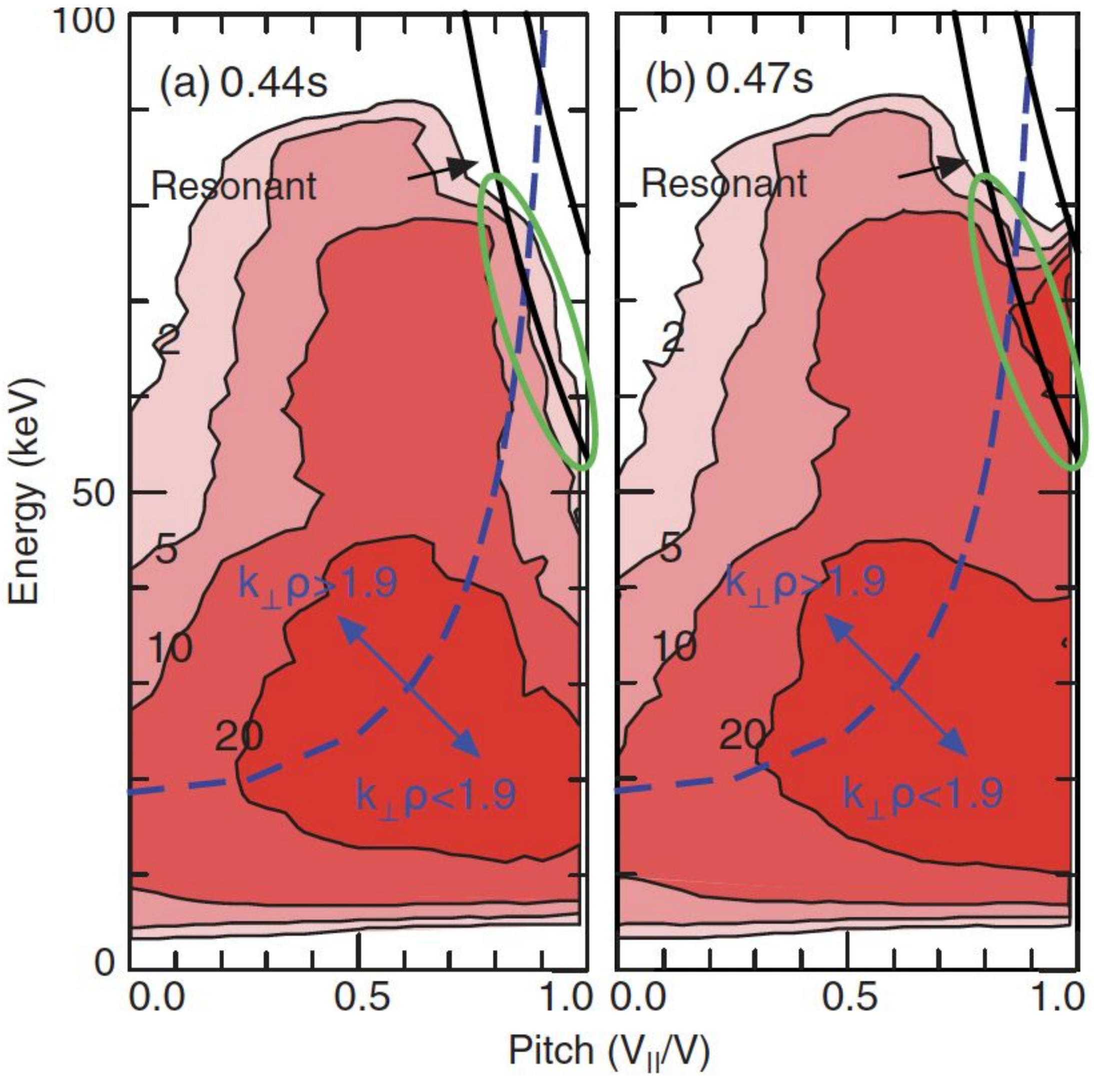
- Use different combinations of on-axis and off-axis beams to vary gradient
- RSAEs are virtually stable for off-axis injection



Heidbrink NF 53 (2013)

The distribution on the left drove instability but the one on the right did not. Which gradient term drove the waves unstable?

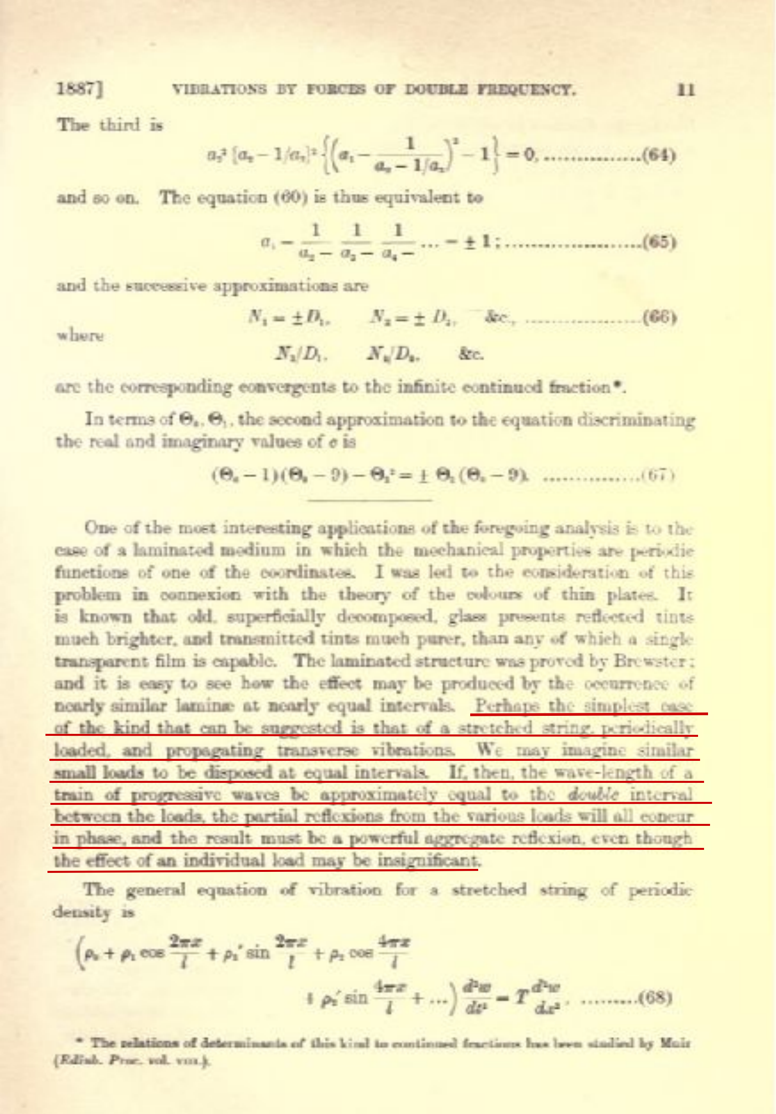
NSTX-U Beam Injection



Fredrickson, PRL 118 (2017) 265001

1. $\partial f/\partial W$ Energy
2. $\partial f/\partial P_\phi$ Radius
3. $\partial f/\partial \mu$ Anisotropy

Periodic index of refraction □ a frequency gap



1887] VIBRATIONS BY FORCES OF DOUBLE FREQUENCY. 11

The third is

$$a_3^2\,[a_2 - 1/a_3]^2 \left\{\left(a_1 - \frac{1}{a_2 - 1/a_3}\right)^2 - 1\right\} = 0, \quad \text{......(64)}$$

and so on. The equation (60) is thus equivalent to

$$a_1 - \frac{1}{a_2 -}\,\frac{1}{a_3 -}\,\frac{1}{a_4 -}\,\ldots = \pm 1; \quad \text{......(65)}$$

and the successive approximations are

$$N_1 = \pm D_1, \qquad N_2 = \pm D_2, \qquad \text{\&c.,} \quad \text{......(66)}$$

where

$$N_1/D_1, \qquad N_2/D_2, \qquad \text{\&c.}$$

are the corresponding convergents to the infinite continued fraction*.

In terms of Θ_0, Θ_1, the second approximation to the equation discriminating the real and imaginary values of c is

$$(\Theta_0 - 1)(\Theta_0 - 9) - \Theta_1^2 = \pm\, \Theta_1 (\Theta_0 - 9). \quad \text{......(67)}$$

One of the most interesting applications of the foregoing analysis is to the case of a laminated medium in which the mechanical properties are periodic functions of one of the coordinates. I was led to the consideration of this problem in connexion with the theory of the colours of thin plates. It is known that old, superficially decomposed, glass presents reflected tints much brighter, and transmitted tints much purer, than any of which a single transparent film is capable. The laminated structure was proved by Brewster; and it is easy to see how the effect may be produced by the occurrence of nearly similar laminæ at nearly equal intervals. Perhaps the simplest case of the kind that can be suggested is that of a stretched string, periodically loaded, and propagating transverse vibrations. We may imagine similar small loads to be disposed at equal intervals. If, then, the wave-length of a train of progressive waves be approximately equal to the *double* interval between the loads, the partial reflexions from the various loads will all concur in phase, and the result must be a powerful aggregate reflexion, even though the effect of an individual load may be insignificant.

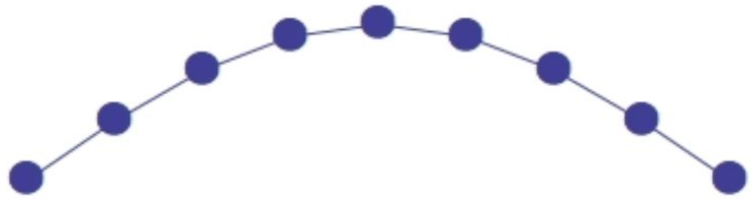
The general equation of vibration for a stretched string of periodic density is

$$\left(\rho_0 + \rho_1 \cos\frac{2\pi x}{l} + \rho_1' \sin\frac{2\pi x}{l} + \rho_2 \cos\frac{4\pi x}{l} + \rho_2' \sin\frac{4\pi x}{l} + \ldots\right)\frac{d^2 w}{dt^2} = T\frac{d^2 w}{dx^2}, \quad \text{......(68)}$$

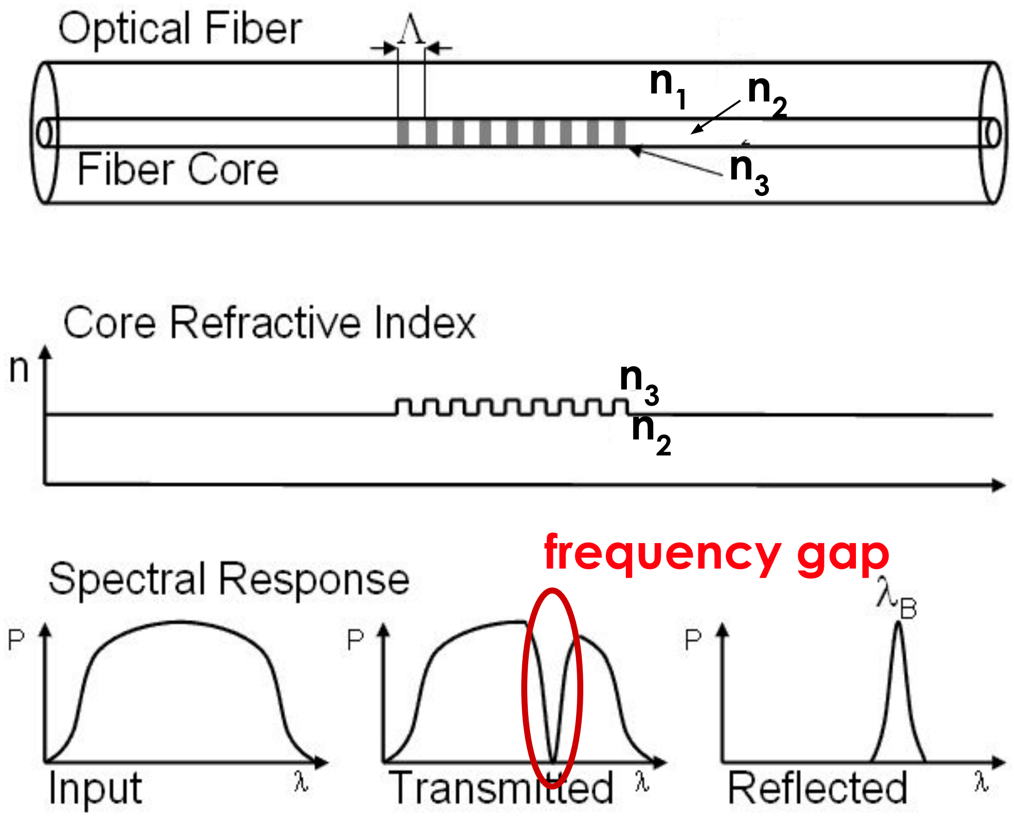
* The relations of determinants of this kind to continued fractions has been studied by Muir (*Edinb. Proc.* vol. VIII.).

Lord Rayleigh, Phil. Mag. (1887)

"Perhaps the simplest case … is that of a stretched string, periodically loaded, and propagating transverse vibrations. …If, then, the wavelength of a train of progressive waves be approximately equal to the *double* interval between the beads, the partial reflexions from the various loads will all concur in phase, and the result must be a powerful aggregate reflexion, even though the effect of an individual load may be insignificant."



The propagation gap occurs at the Bragg frequency & its width is proportional to ΔN



Wikipedia, "Fiber Bragg grating"

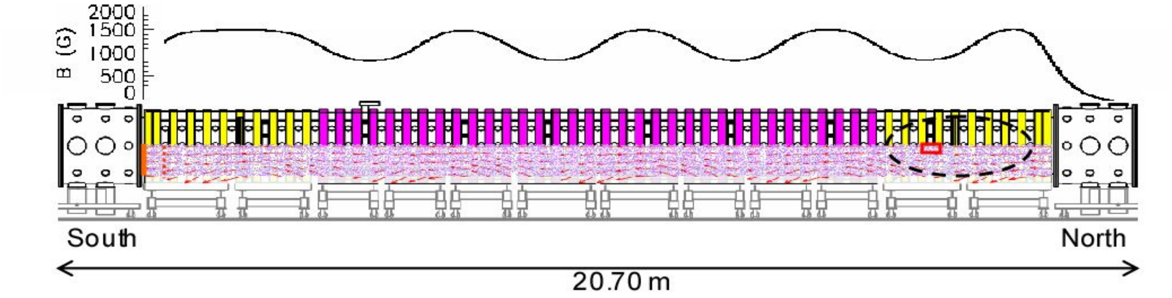
- **Destructive interference between counter propagating waves**
- **Bragg frequency: $f=v/2\Lambda$**
- **$\Delta f/f \sim \Delta N/N$**

for shear Alfvén waves

- **$f = v_A / 2\Lambda$, where Λ is the distance between field maxima along the field line**
- **$\Delta f \sim \Delta B/B$**

Periodic variation of the magnetic field produces periodic variations in N for shear Alfvén waves

Periodic Mirror Field in the LAPD



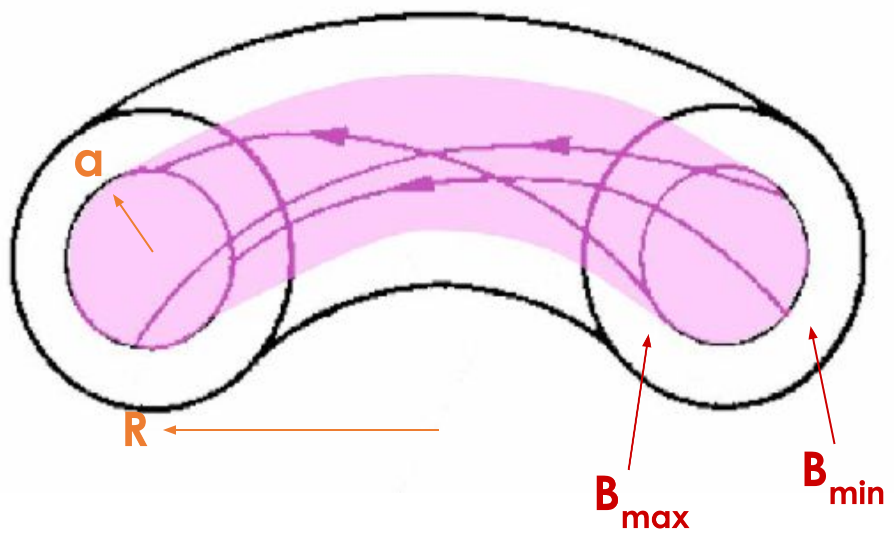
Zhang, Phys. Plasmas 14 (2007)

Periodic variation in B □
Periodic variation in v_A □
Periodic variation in index of refraction N

□ Frequency gap that is proportional to ΔN

Frequency gaps are unavoidable in a toroidal confinement device

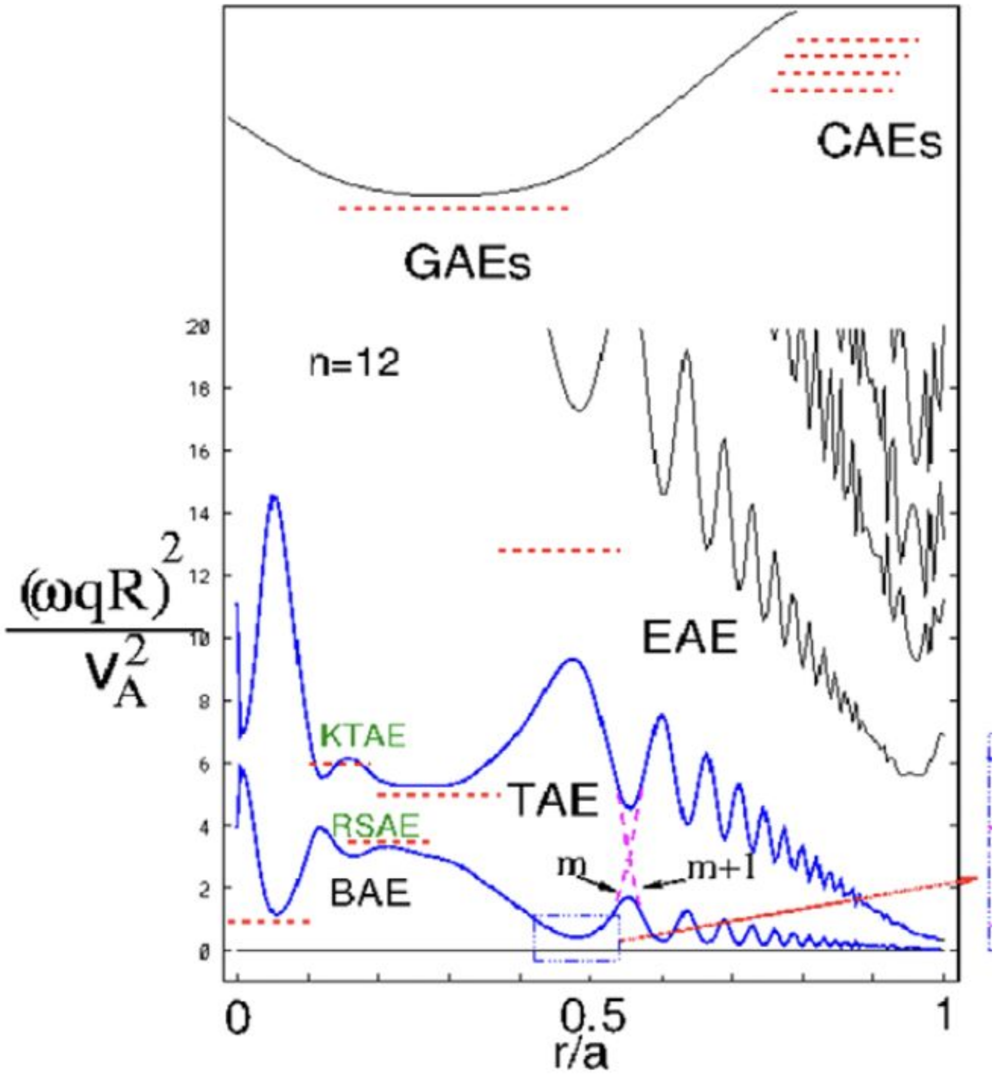
Field lines in a tokamak



Example for "toroidicity-induced Alfvén eigenmode (TAE)

- $B \sim 1/R$
- $\Delta B \sim a/R$
- Distance between maxima is $\Lambda = q\,(2\pi R)$ so $f_{gap} = v_A/4\pi qR$

All periodic variations produce frequency gaps



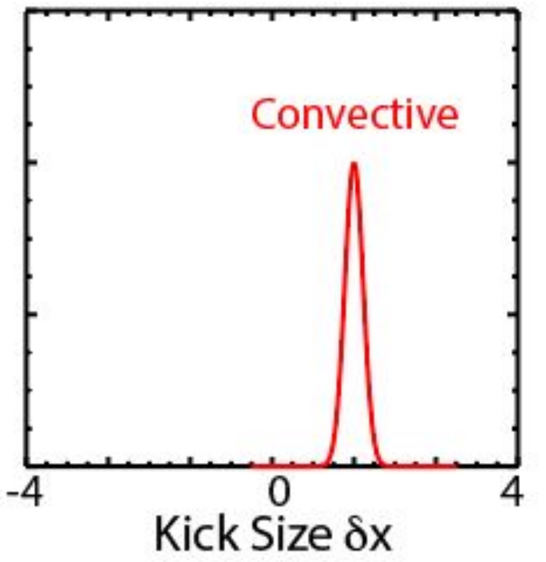
Gorelenkov, Nucl. Fusion 54 (2014) 125001

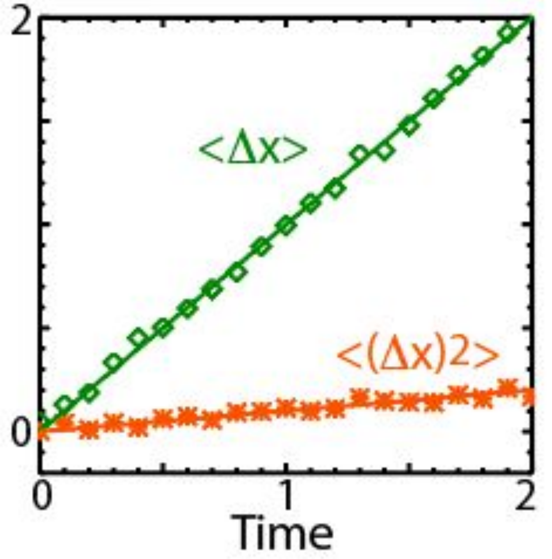
BAE	"beta"	compression
TAE	"toroidicity"	m & m+1
EAE	"ellipticity"	m & m+2
NAE	"noncircular"	m & m+3
MAE	"mirror"	n & n+1
HAE	"helicity"	both n's & m's

Outline

1. <u>Orbits</u> are described by constants of motion
2. <u>Perturbations</u> break constants-of-motion & cause transport
3. The wave-particle <u>phase</u> determines the type of transport
4. For non-resonant modes, <u>orbit averaging</u> dramatically reduces transport
5. Resonant particles can drive <u>instabiity</u>

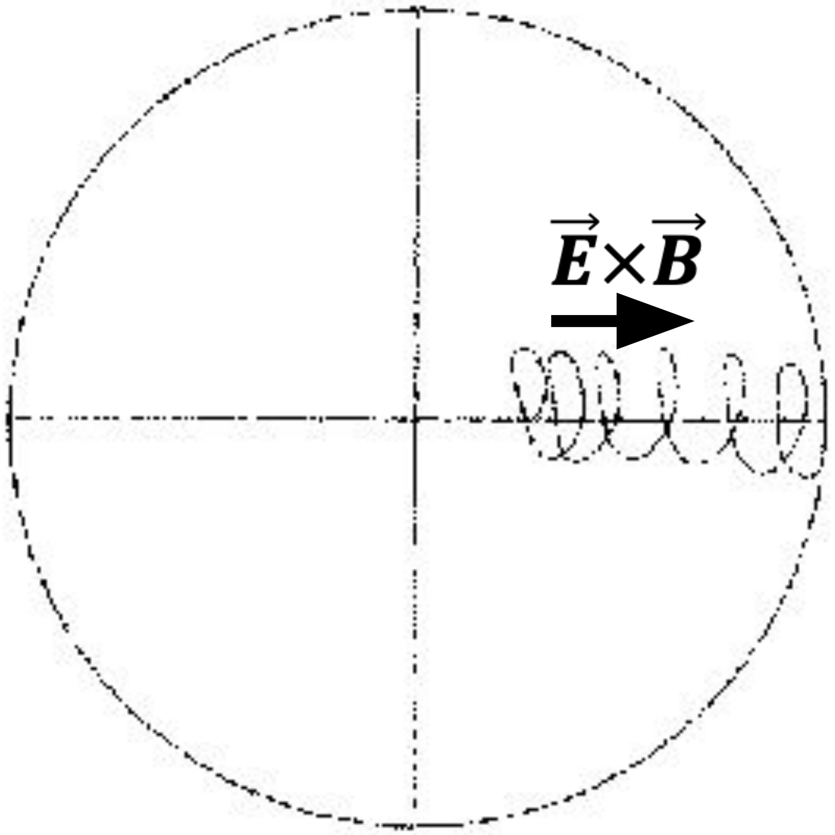
6. An EP that stays in phase with a single mode experiences large <u>convective resonant transport</u>





Coherent convective transport occurs for modes that maintain resonance across the plasma

Calculated guiding-center loss orbit caused by a "fishbone" in the PDX tokamak



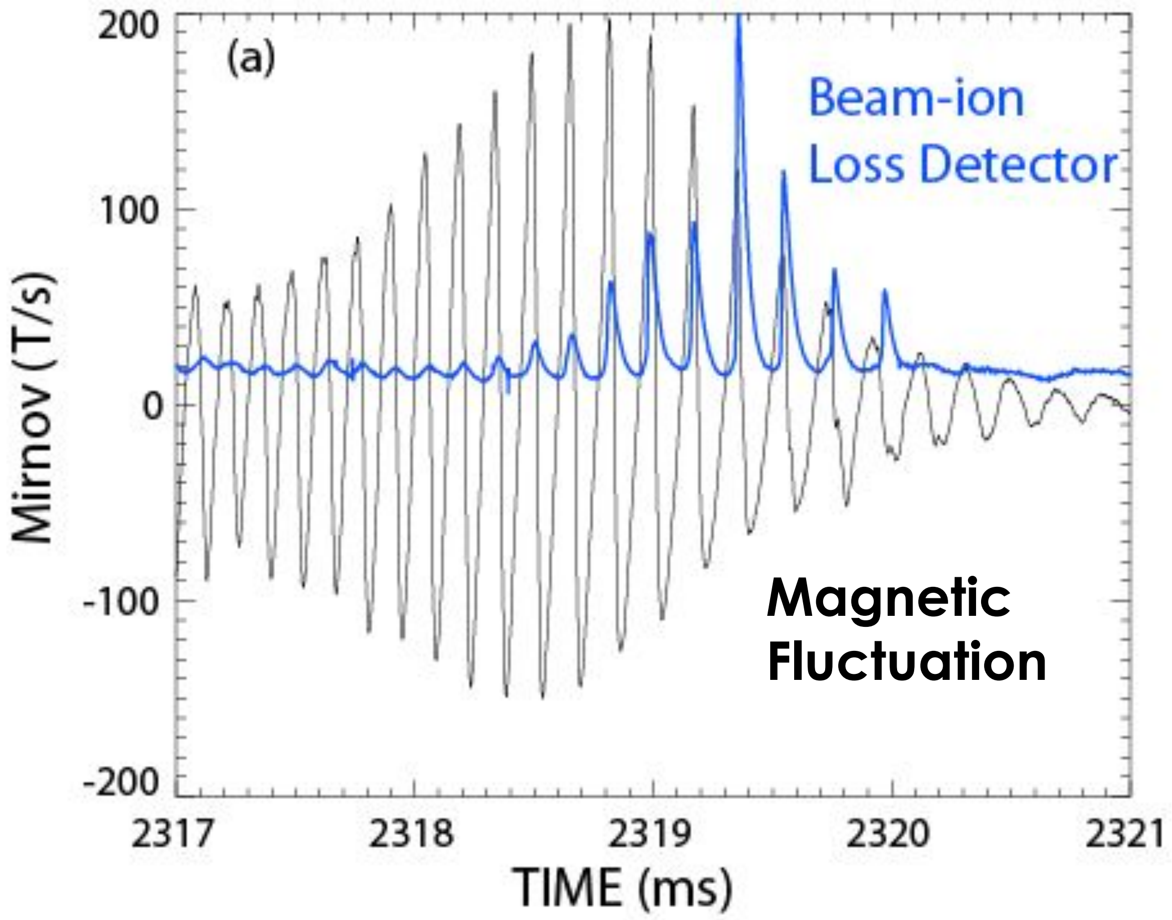
White, Phys. Fluids 26 (1983) 2958

- The fishbone was a globally extended, low-frequency mode
- Low frequency → 1st & 2nd adiabatic invariants are conserved
- The mode extracts energy from the fast ions → fast-ion energy decreases
- μ conservation → $W_{\perp}/B$ = constant → particle must move out (to lower B)
- Main loss mechanism: convective E x B radial transport
- As expected, lost ions lose ~ 10 keV

Beiersdorfer, Nucl. Fusion 24 (1984) 487

Losses have a definite phase relative to the mode

DIII-D tokamak off-axis fishbone data



Heidbrink, Plasma Phys. Cont. Fusion 53 (2011) 085028

- **Particles are expelled in a "lighthouse beacon" that rotates with the mode**
- **Losses occur at the phase that pushes particles outward**

Losses scale linearly with mode amplitude

Beam ion flux to loss detector on CHS stellarator during an Energetic Particle Mode

- **Flux ∝ $\vec{E}\times\vec{B}$ → coherent loss rate scales linearly with mode amplitude**
- **Similar result for RF tail ions expelled by Alfven eigenmodes in the ASDEX-Upgrade tokamak**

Garcia-Muñoz, PRL 104 (2010) 185002

Nagaoka, PRL 100 (2008) 065005

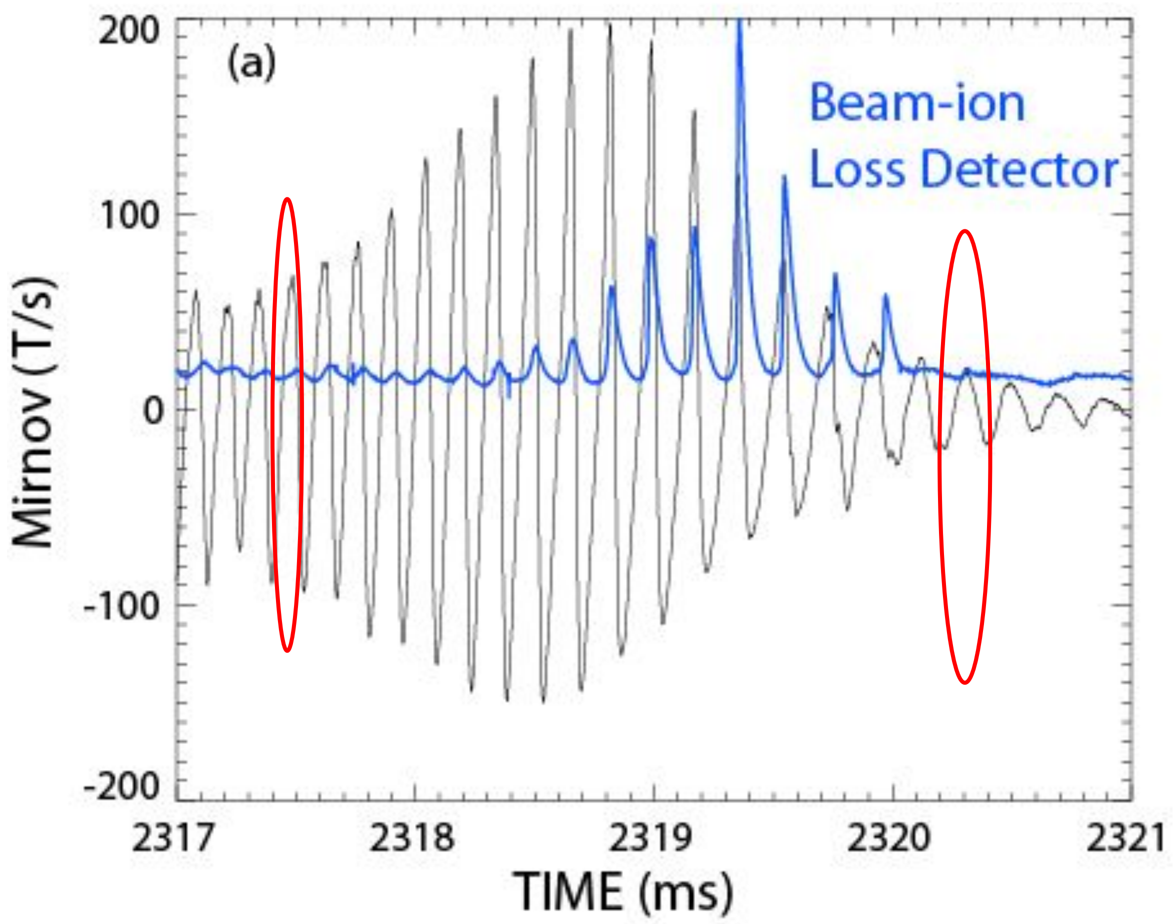
Energetic Particle Modes (EPM) are a type of "beam mode"

Normal Mode (gap mode)	Energetic Particle Mode[1]
$n_{EP} << n_e$	$\beta_{EP} \sim \beta$
Wave exists w/o EPs.	EPs create a new wave branch
Re(ω) unaffected by EPs.	Re(ω) depends on EP distrib. function
EPs resonate with mode, altering Im(ω)	EPs resonate with mode, altering Im(ω)
Gap mode avoids continuum damping	Intense drive overcomes continuum damping

Chen, Phys. Plasma 1 (1994) 1519

In an EPM, the frequency often "chirps" to maintain resonance with the driving EPs

<u>DIII-D tokamak off-axis fishbone data</u>



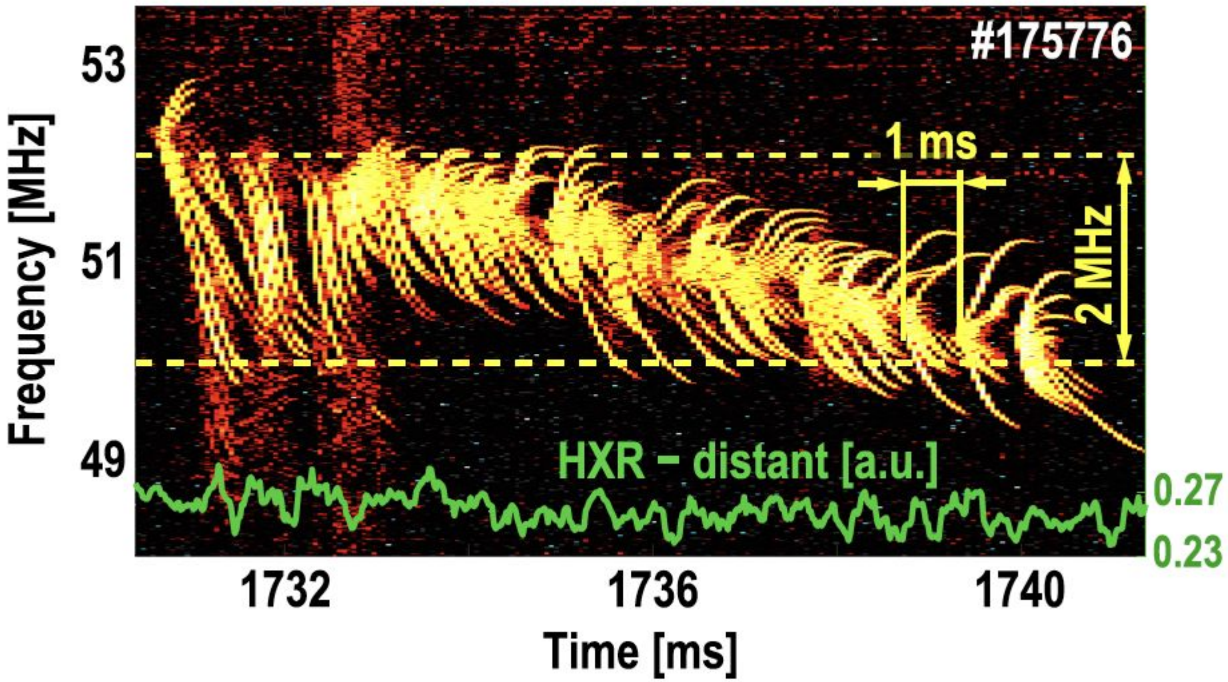
Heidbrink, Plasma Phys. Cont. Fusion 53 (2011) 085028

EP terminology distinguishes between frequency “chirping” & frequency “sweeping”

Energetic Particle Mode

- Re(ω) tracks changes in distribution function (millisecond timescale)

Runaway driven chirping modes on DIII-D

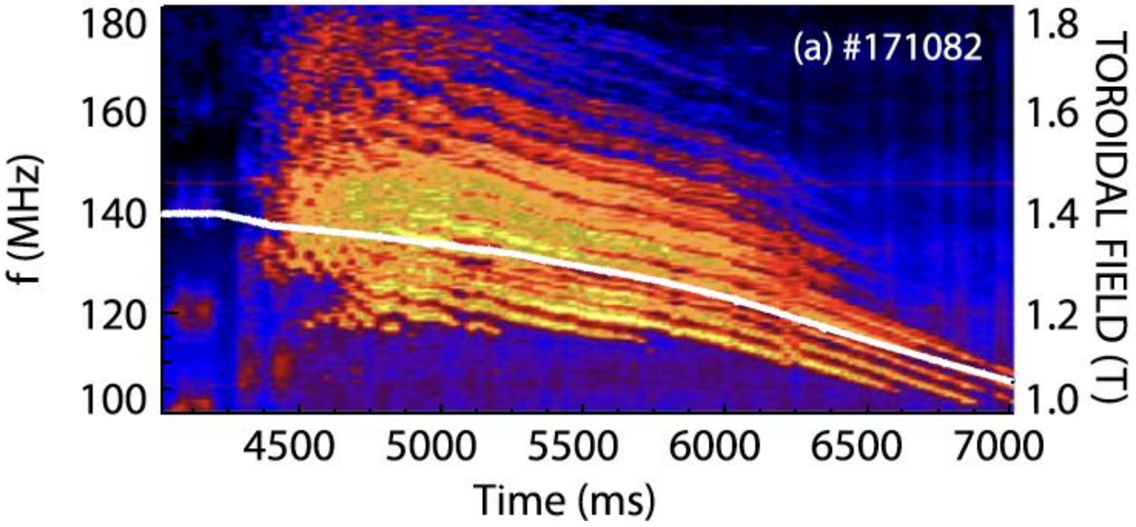


Lvovskiy, Nucl. Fusion 59 (2019) 124004

Normal Mode (gap mode)

- Re(ω) tracks changes in plasma parameters (equilibrium timescale)

Runaway driven whistler waves on DIII-D



Heidbrink PPCF 61 (2019) 014007

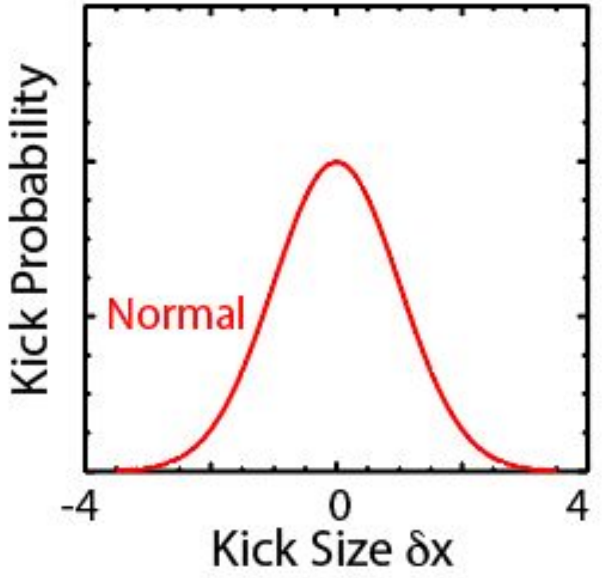
Note: Normal modes can also chirp

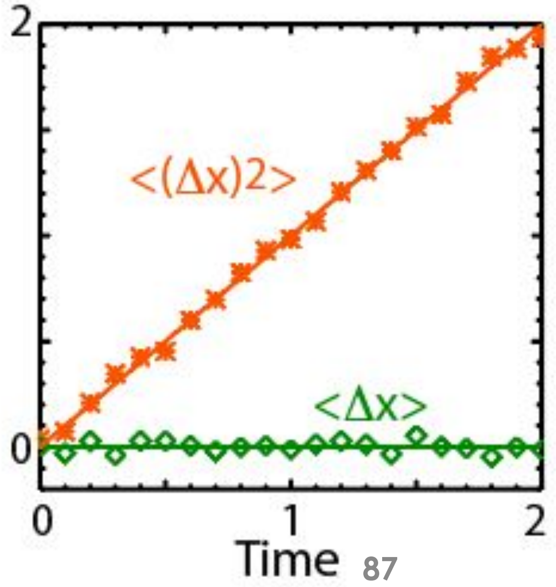
Outline

1. Orbits are described by constants of motion
2. Field perturbations break constants-of-motion & cause transport
3. The wave-particle phase determines the type of transport
4. For non-resonant modes, orbit averaging dramatically reduces transport
5. Resonant particles can drive instability
6. An EP that stays in phase with a single mode experiences large convective resonant transport

7. Multiple resonances cause diffusive & stiff transport

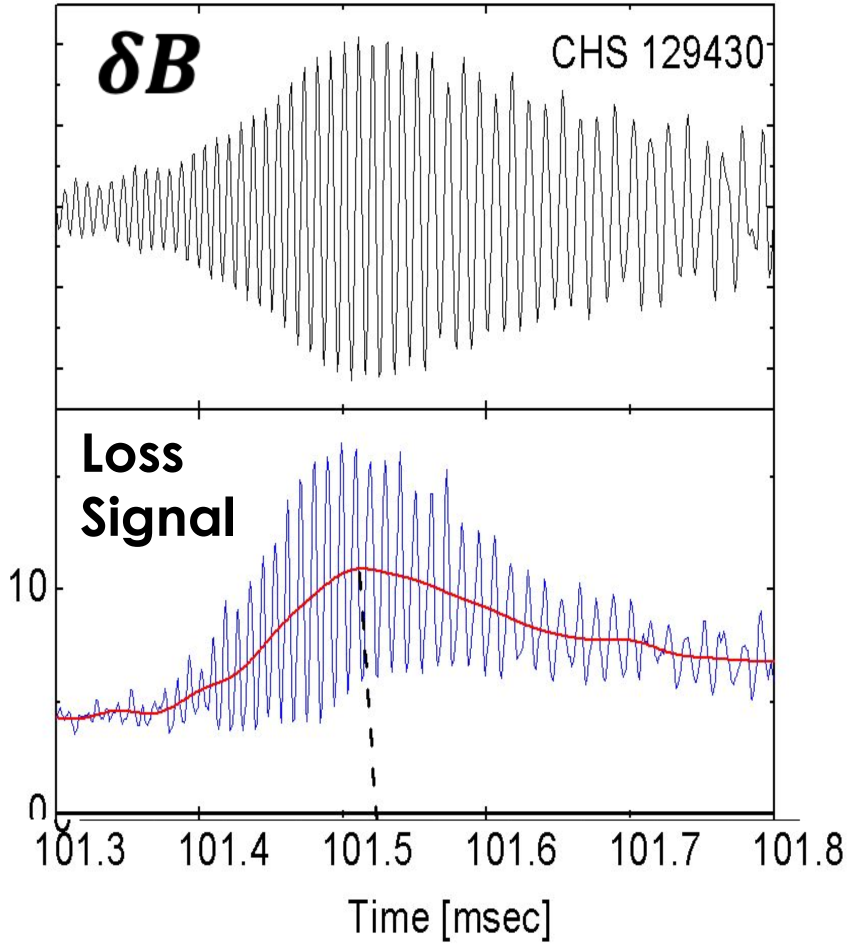
- Large orbits can cause multiple resonances with a single wave
- Diffusive transport often scales quadratically with mode amplitude
- Stochastic thresholds can cause stiff transport





Losses scale linearly with mode amplitude
FOR CONVECTIVE RESONANT TRANSPORT

Beam ion flux to loss detector on CHS stellarator during an Energetic Particle Mode



Nagaoka, PRL 100 (2008) 065005

Losses scale quadratically with mode amplitude for diffusive resonant transport

Beam ion flux to loss detector on CHS stellarator during an Energetic Particle Mode

Convective

Diffusive

Nagaoka, PRL 100 (2008) 065005

- Similar result on ASDEX-Upgrade
  Garcia-Munoz, PRL 104 (2010) 185002

Losses scale quadratically with mode amplitude for diffusive resonant transport

- $\delta P_\phi \propto \delta B$ (kick in momentum)
- $D \propto (\delta x)^2$ (random walk)
- Flux $\Gamma \propto -D\nabla n$ (Fick's law)

→ Flux ∝ (Amplitude)²

Hsu, Phys. Fluids B 4 (1992) 1492

Beam ion flux to loss detector on CHS stellarator during an Energetic Particle Mode

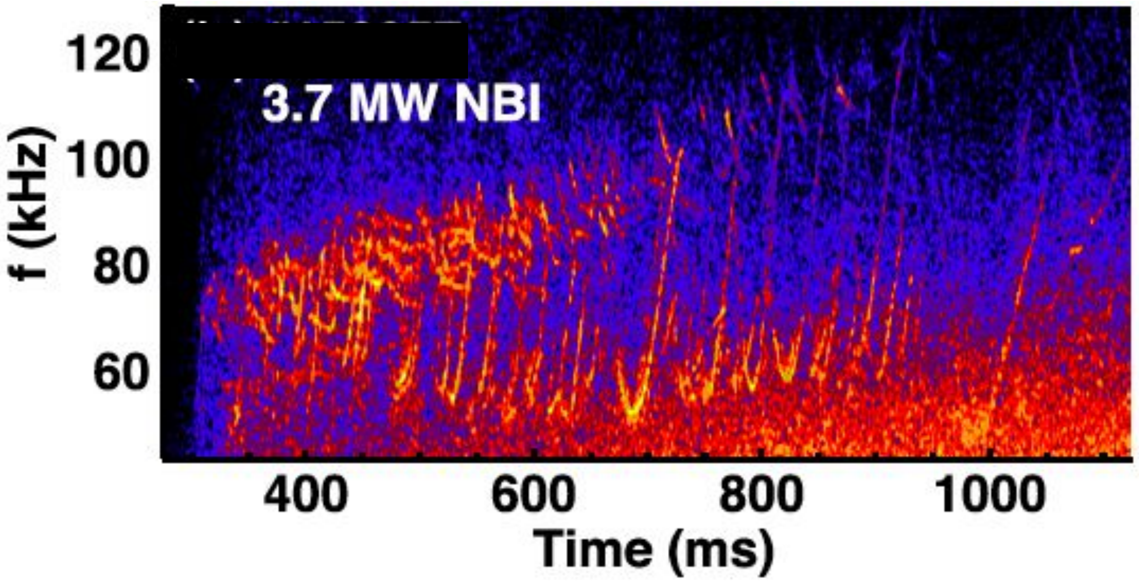
Diffusive

Crossing a stochastic threshold
 different scaling

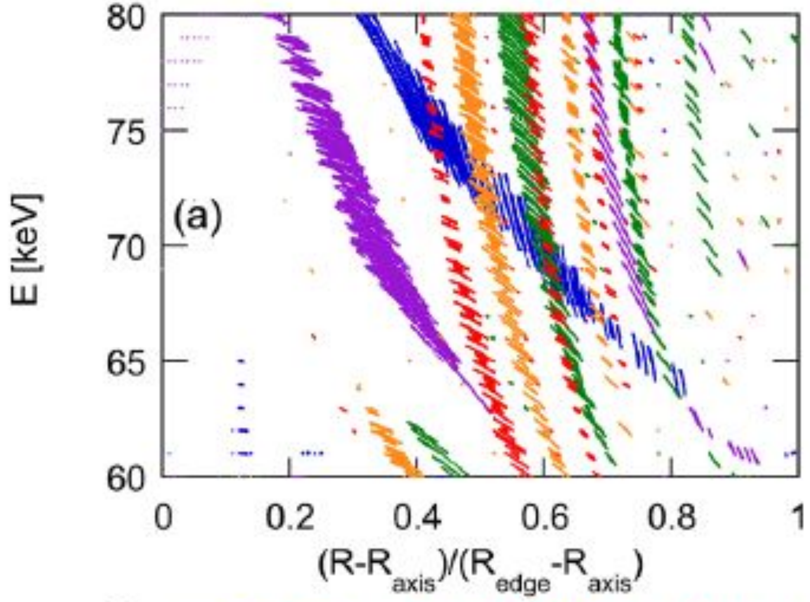
- Similar result on ASDEX-Upgrade
  Garcia-Munoz, PRL 104 (2010) 185002

Many small-amplitude resonances □ stochastic threshold ("stiff" transport)

Measured Alfven eigenmodes in the DIII-D tokamak



Calculated resonances in phase space



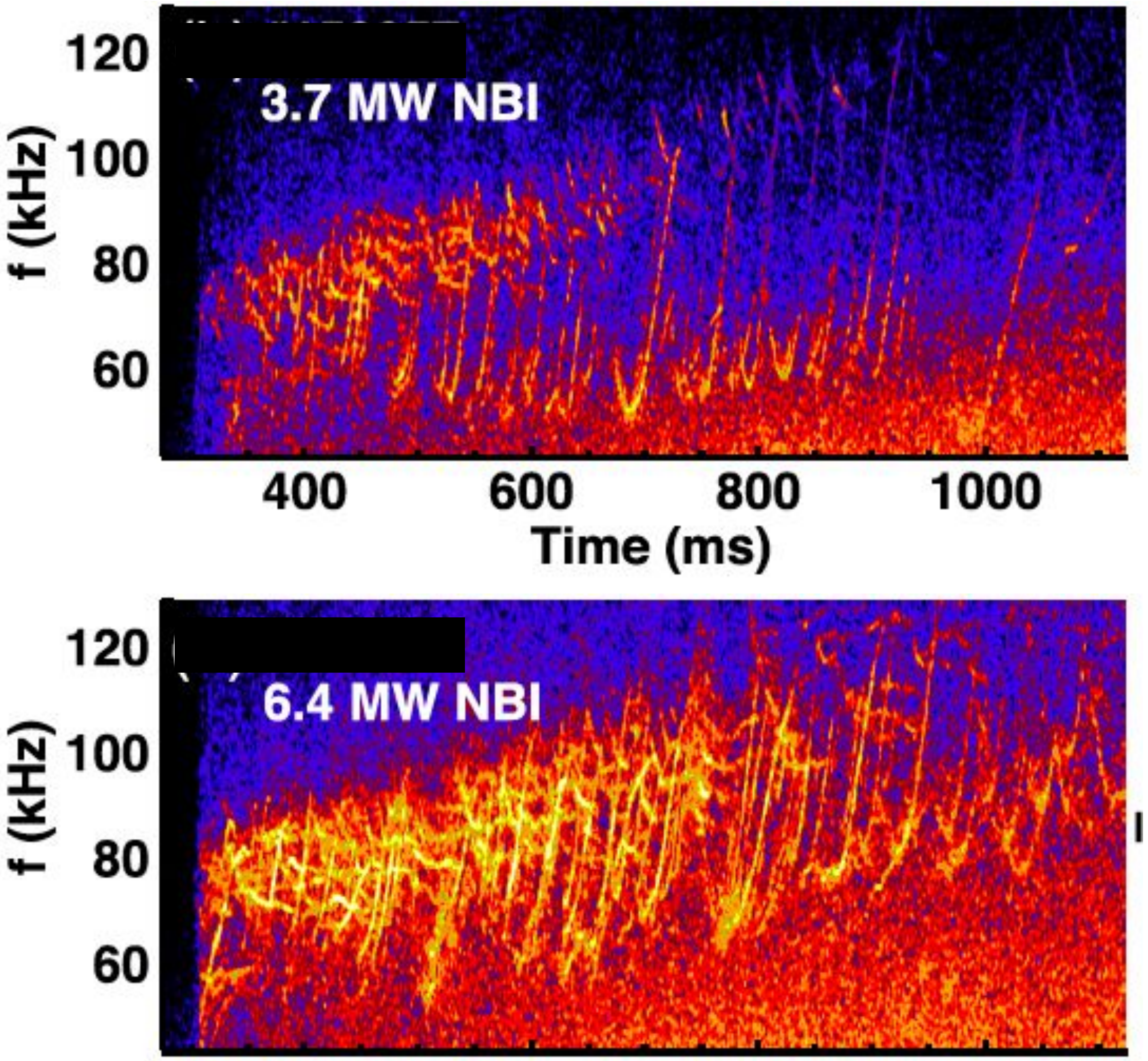
Little transport

Collins, PRL 116 (2016) 095001

Todo, Nucl. Fusion 56 (2016) 112008

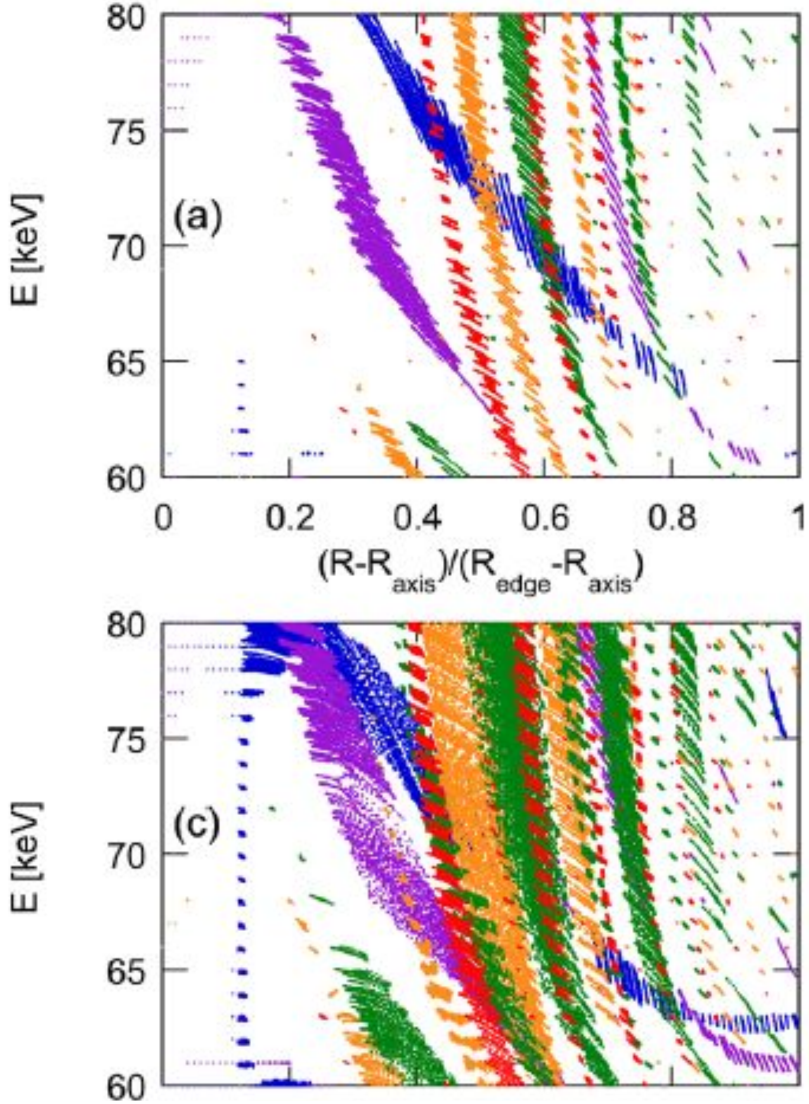
Many small-amplitude resonances □ stochastic threshold (“stiff” transport)

Measured Alfven eigenmodes in the DIII-D tokamak



Collins, PRL 116 (2016) 095001

Calculated resonances in phase space



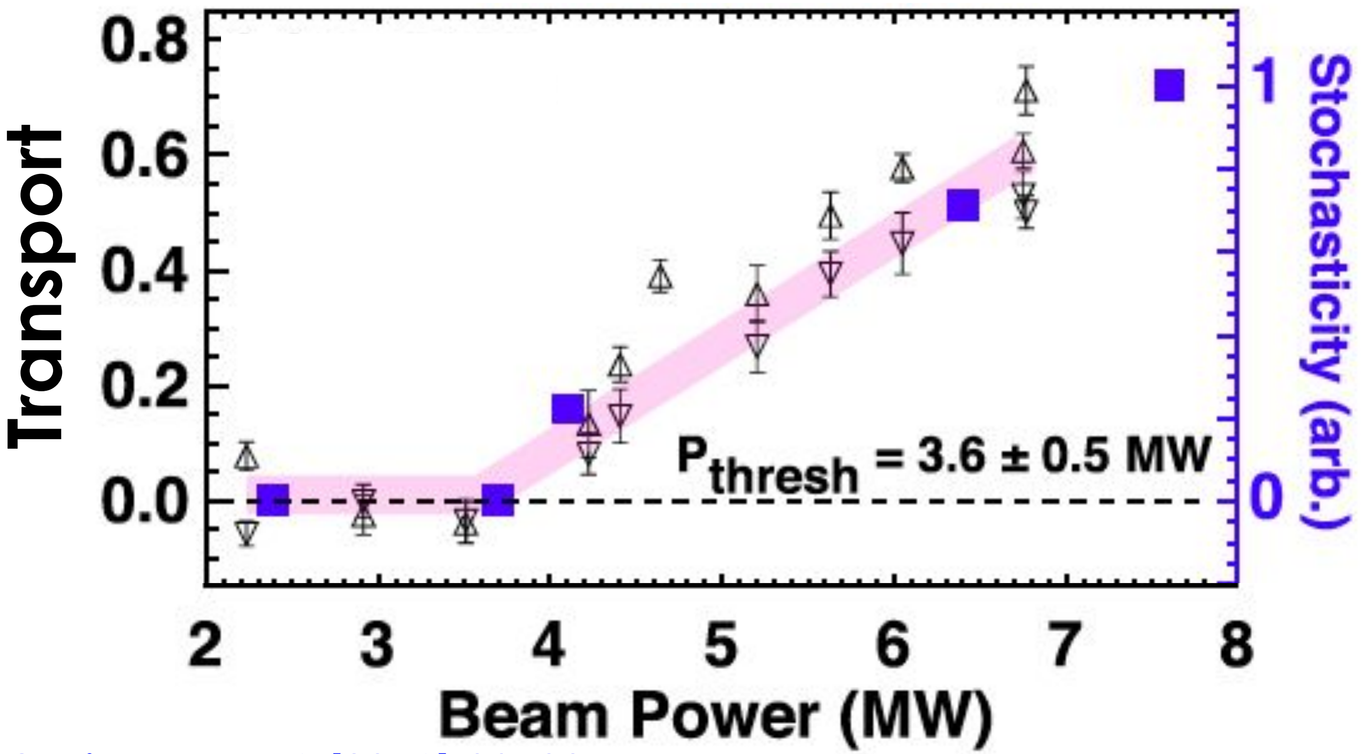
Little transport

Large transport

Todo, Nucl. Fusion 56 (2016) 112008

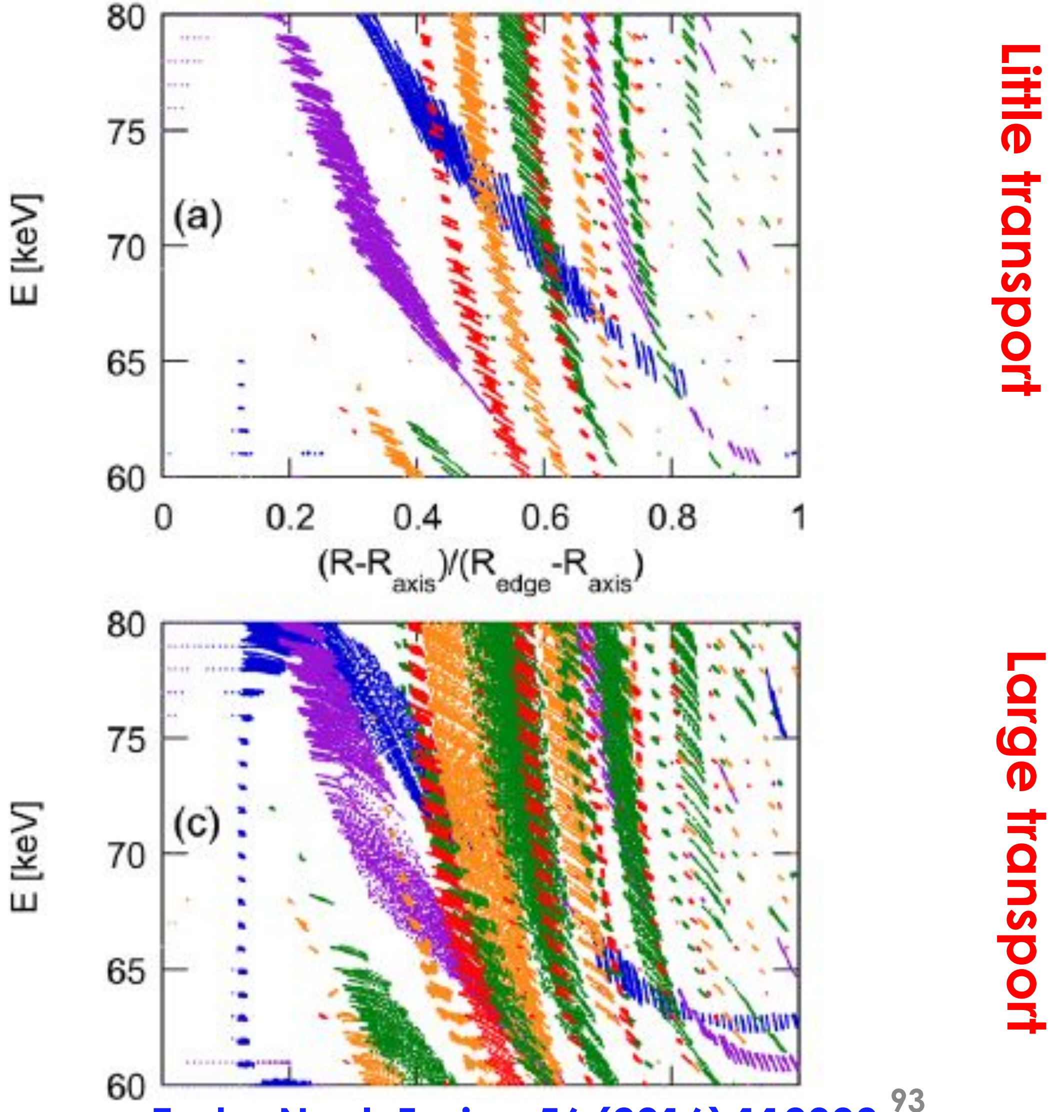
Many small-amplitude resonances stochastic threshold ("stiff" transport)

Measured "stiff" fast-ion transport in the DIII-D tokamak



Collins, PRL 116 (2016) 095001

Calculated resonances in phase space



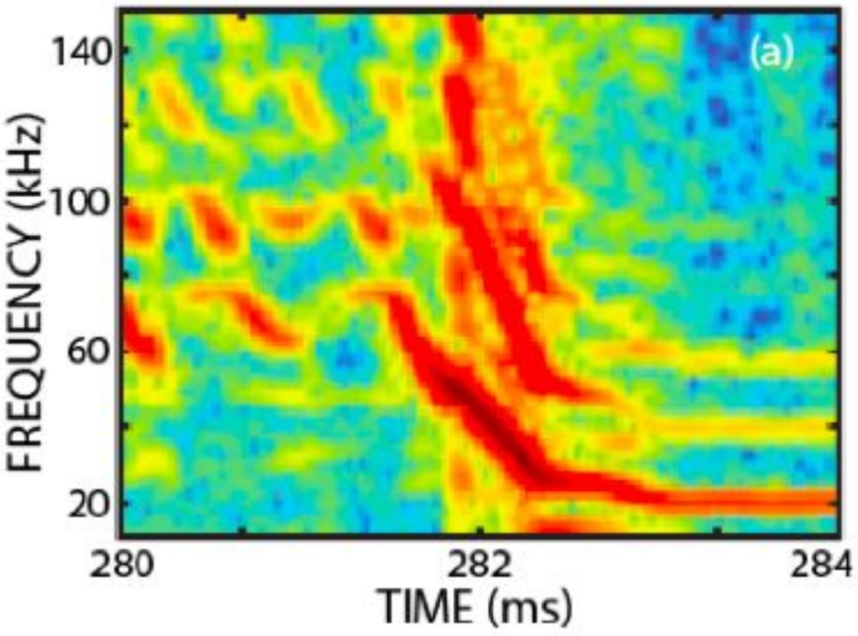
Todo, Nucl. Fusion 56 (2016) 112008

For EP-driven instabilities, the stochastic threshold is often crossed transiently

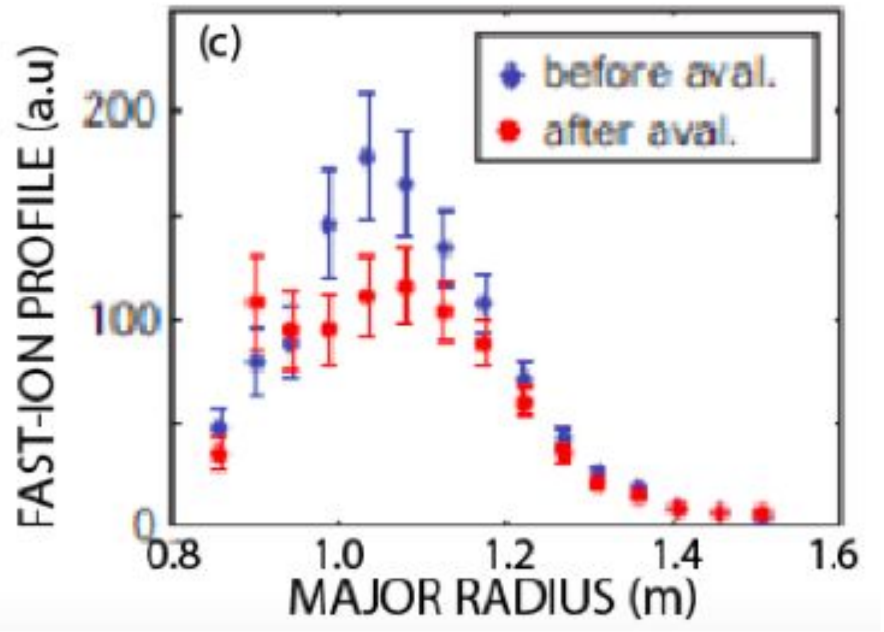
- More modes & larger amplitudes resonances overlap
- Similar phenomena in JT-60U

Ishikawa, Nucl. Fusion 45 (2005) 1474; Bierwage, Nucl. Fusion 57 (2017) 016036

Modes in the NSTX spherical tokamak



Measured beam-ion profiles



Podesta, Nucl. Fusion 56 (2016) 112005

Which mechanism can cause stochastic EP loss orbits in tokamaks?

1. Scattering off of drift wave turbulence
2. Pitch-angle scattering across a topological boundary
3. Overlap of island chains in orbit phase space in the presence of a large (2,1) tearing mode
4. Staying in phase with a chirping fishbone
5. A large number of small amplitude Alfvén eigenmodes
6. None of these
7. Two of these
8. Three of these

Outline

1. Orbits are described by constants of motion[1]
2. Coulomb collisions[2] & field perturbations[1] break constants-of-motion & cause transport
3. The wave-particle phase determines resonance[1]
4. For non-resonant modes, orbit averaging dramatically reduces transport[1]
5. Resonances can drive instability[3]
6. An EP that stays in phase with a single mode experiences large convective resonant transport[1]
7. Multiple resonances cause diffusive & stiff transport[1]

1. "Mechanisms of energetic-particle transport in magnetically confined plasma," Phys. Pl. 27 (2020) 030901
2. "The behaviour of fast ions in tokamak experiments,", Nucl. Fusion 34 (1994) 535.
3. "Basic physics of Alfvén instabilities driven by energetic particles in toroidally confined plasmas," Phys. Pl. 15 (2008) 055501